Isospin-symmetry-breaking effects in nuclear $\beta$ decay

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1. **CKM unitarity and pure Fermi decays**
   - **introduction/motivation** – CKM unitarity and $0^+ \rightarrow 0^+$ $\beta$ decays
   - **isospin-symmetry-breaking (ISB) effects** in nuclei
   - **current status** – theoretical corrections questioned
1. **CKM unitarity and pure Fermi decays**
   - introduction/motivation – CKM unitarity and $0^+ \rightarrow 0^+$ $\beta$ decays
   - isospin-symmetry-breaking (ISB) effects in nuclei
   - current status – theoretical corrections questioned

2. **Tests of ISB corrections**
   - existing 13 most precise pure Fermi transitions
   - $T = 2$ $\beta$-delayed proton decay of $^{32}$Ar
   - $T = 1$ superallowed branch of $^{32}$Cl
What is the “CKM matrix”? 

Cabibbo

mass eigenstates \( \neq \) weak eigenstates

\[ |d'\rangle = \cos \theta_C |d\rangle + \sin \theta_C |s\rangle \]
\[ |s'\rangle = -\sin \theta_C |d\rangle + \cos \theta_C |s\rangle \]

flavour mixing

purely leptonic

\[ \sqrt{G_F} \]
\[ W^+ \]
\[ \mu^+ \]
\[ \nu_\mu \]
\[ \sqrt{G_F} = \text{strength of lepton coupling to } W \]

semileptonic

\[ \sqrt{G_F} \cos \theta_C \]
\[ W^+ \]
\[ u \]
\[ d \]
\[ \sqrt{G_F} \cos \theta_C = \text{strength of quark coupling to } W \]
What is the “CKM matrix”?  

**Cabibbo**

mass eigenstates ≠ weak eigenstates

**flavour mixing**

\[
\begin{align*}
|d'\rangle &= \cos \theta_C |d\rangle + \sin \theta_C |s\rangle \\
|s'\rangle &= -\sin \theta_C |d\rangle + \cos \theta_C |s\rangle
\end{align*}
\]

“**Unitarity**” ⇔ \( \cos \theta_C^2 + \sin \theta_C^2 = 1 \)

(if we have the physics right...)

**purely leptonic**

\[
\sqrt{G_F} W^+ \rightarrow e^+ \nu_e \\
\mu^+ \rightarrow \nu_\mu
\]

\( \sqrt{G_F} = \text{strength of lepton coupling to } W \)

**semi-leptonic**

\[
\sqrt{G_F} \cos \theta_C \rightarrow \nu_e \\

u \rightarrow d \\
\sqrt{G_F} \cos \theta_C = \text{strength of quark coupling to } W
\]
What is the “CKM matrix”?

mass eigenstates ≠ weak eigenstates

generalized Cabibbo’s theory to three generations

\[ \left( \begin{array}{c} d' \\ s' \\ b' \end{array} \right) = \left( \begin{array}{ccc} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{array} \right) \left( \begin{array}{c} d \\ s \\ b \end{array} \right) \]

\[ V_{ud} = \frac{G_V}{G_F} \]
The unitarity test

\[
\begin{pmatrix}
  d' \\
  s' \\
  b'
\end{pmatrix}

= \begin{pmatrix}
  0.974 & 0.225 & 0.004 \\
  0.224 & 0.973 & 0.04 \\
  0.01 & 0.04 & 0.999
\end{pmatrix}
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}
\]

weak eigenstates

mass eigenstates

\[|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1?\]

\[|V_{ud}| = 0.97425(22) \quad \text{Towner & Hardy Rep.}\]
\[|V_{us}| = 0.22521(94) \quad \text{Prog. Phys. 73 (2010)}\]
\[|V_{ub}| = 0.00393(36) \quad \text{(is negligible)}\]

\[\sum |V_{u,i}|^2 = 0.99990(60)\]

0.06% \Rightarrow \text{constrains new physics}
Impressive agreement with unitarity shows no sign of new physics, but does help constrain extensions to the standard model.

Examples include (with some caveats we won’t go into):

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Examples include (with some caveats we won’t go into):

- right-handed currents: $\sum |V_{u,i}|^2 = 1 + 2\Re(e^{\alpha_{LR}})$

  \[\Rightarrow \Re(e^{\alpha_{LR}}) = (-5 \pm 30) \times 10^{-5}\]

  (cf $\mu$ decay: $|\Re(e^{\alpha_{LR}})| \lesssim 2300 \times 10^{-5}$)
Impressive agreement with unitarity shows **no sign** of new physics, but *does* help constrain extensions to the standard model.

Examples include (with some caveats we won’t go into):

- **right-handed currents:** \( \mathcal{Re}(\bar{a}_{LR}) = (-5 \pm 30) \times 10^{-5} \)
- **scalar currents:** \( |C_s/C_V| \leq 0.065 \)
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Examples include (with some caveats we won’t go into):

- **right-handed currents**: \( \Re(a_{LR}) = (-5 \pm 30) \times 10^{-5} \)
- **scalar currents**: \( |C_S/C_V| \leq 0.065 \)
- **extra \( Z \) bosons**: \( M_{Z_X} > 460 \text{ GeV} \)
- **4\(^{\text{th}}\) generation of quarks**
- **exotic muon decay**
- **supersymmetry**
\[ \beta \text{ transitions between } 0^+ \text{ states are theoretically well-understood within the Standard Model:} \]

\[
\left( \text{phase space} \right) \left( \text{partial half-life} \right) \equiv f t = \frac{K/G_F^2}{V_{ud}^2 (M_F^2 + M_{GT}^2) (1 + \Delta V_R)}
\]

\[
f \equiv \int F(Z, E) C(E) p E (E - E_0)^2 dE \\
\sim Q_{EC}^5
\]

\[
t \equiv \frac{t_{1/2}}{Br} (1 + P_{EC})
\]
$\beta$ transitions between $0^+$ states are theoretically well-understood within the Standard Model:

\[
\left( \text{phase space} \right) \left( \text{partial half-life} \right) \equiv f_t = \frac{K/G_F^2}{V_{ud}^2(M^2_F + M^2_{GT})(1 + \Delta^V_R)}
\]

\[
K/(\hbar c)^6 = \frac{2\pi^3\hbar \ln 2}{(m_e c^2)^5}
\]

is a pure constant,

\[
G_F = 1.16639(1) \times 10^{-5} \text{ GeV}^{-2}
\]
from muon decay.
Pure Fermi Transitions

$\beta$ transitions between $0^+$ states are theoretically well-understood within the Standard Model:

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(\text{phase space}) (\text{partial half-life}) \equiv f_t = \frac{K/G_F^2}{V_{ud}^2(M_F^2 + M_{GT}^2)(1 + \Delta_V^R)} \rightarrow \text{5968.7(1) s}
\]

\[
M_{GT} = 0 \quad \text{since this is } 0^+ \rightarrow 0^+
\]
\( \beta \) transitions between \( 0^+ \) states are theoretically well-understood within the Standard Model:

\[
\begin{align*}
(\text{phase space})(\text{partial half-life}) & \equiv ft = \frac{K/G_F}{\mathcal{V}_{ud}^2(M_F^2 + M_{GT}^2)(1 + \Delta_V^R)} \\
M_F & \approx \langle T \ T_Z \pm 1 | \tau_\pm | T \ T_Z \rangle = \sqrt{T(T+1) - T_Z(T_Z \pm 1)} \\
& = \sqrt{2} \quad \text{for } T = 1 \text{ transitions}
\end{align*}
\]
$\beta$ transitions between $0^+$ states are theoretically well-understood within the Standard Model:

\[
(\text{phase space})(\text{partial half-life}) \equiv f_t = \frac{K/G_F^2}{V_{ud}^2(M_F^2 + M_{GT}^2)^2(1 + \Delta^V_R)}
\]

$\Delta^V_R = 2.261(38)\%$ is a nucleus-independent radiative correction


$\rightsquigarrow$ currently limits unitarity test — improvement here would \textit{directly} lead to a more stringent test
$\beta$ transitions between $0^+$ states are theoretically well-understood within the Standard Model:

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\[ \approx 5968.7(1) \text{ s} \]

\[ \approx 2.36(4)\% \]

\[ \Rightarrow \text{the level to which this quantity is nucleus-independent is a test of the conserved vector current (CVC) \text{ "hypothesis"}} \]
Pure Fermi Transitions

$\beta$ transitions between $0^+$ states are theoretically well-understood within the Standard Model:

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\left( \text{phase space} \right) \left( \text{partial half-life} \right) \equiv f_t = \frac{K / G_F^2}{V_{ud}^2 \left( M_F^2 + M_{GT}^2 \right) \left( 1 + \Delta V_R \right)} \equiv 5968.7(1) \text{ s}
\]

\[
V_{ud}^2 = \frac{2915.6 \pm 1.1 \text{ s}}{f_t}
\]

$\Rightarrow$ the level to which this quantity is nucleus-independent is a test of the conserved vector current (CVC) “hypothesis”

$\Rightarrow$ if CVC holds, then can use $0^+ \rightarrow 0^+$ decays to measure $V_{ud}$ via:
We must account for the fact that the decay occurs within the nuclear medium.

\[ \mathcal{F}t \equiv ft \left( 1 + \delta'_R \right) \left( 1 + (\delta_{NS} - \delta_C) \right) = \frac{2915.6 \pm 1.1 \text{ s}}{|V_{ud}|^2} \]

(really should be constant)

- \( \delta'_R = E_{e}^{\text{max}} \) and \( Z \) dependent radiative correction
- \( \delta_{NS} \) = nuclear structure dependent radiative correction
- \( \delta_C \) = isospin symmetry-breaking correction
Corrections to the $f_t$ values
Corrections to the $ft$ values

$\Rightarrow \delta'_R$ shifts all values $\approx$ equally
Corrections to the $f t$ values

$\delta_{NS}$ is a small correction

$Z$ of parent nucleus
Corrections to the $ft$ values

$\Rightarrow \delta_C$ is the one that aligns them!
Corrections to the $ft$ values

$\langle Ft \rangle = 3072.08(79) \text{ sec}$

$\chi^2/12 = 0.28$

tests the Standard Model
Corrections to the \( ft \) values

\[ \langle \mathcal{F}t \rangle = 3072.08(79) \text{ sec} \]

\[ \chi^2/12 = 0.28 \]

tests the Standard Model

\[ \approx \] CVC “hypothesis”

\[ \approx \approx \approx \] \( V_{ud} \) and CKM unitarity

\[ \approx \approx \approx \approx \] fundamental scalar currents

\[ \approx \approx \approx \approx \approx \] right-handed currents (via mixing)

\[ \approx \approx \approx \approx \approx \approx \] extra \( Z \) bosons
Emphasis (scrutiny) now is on $\delta_C$

$\delta_C$ corrects the Fermi matrix element for charge-dependent forces:

$$M_F^2 = M_0^2 (1 + \delta_C)$$

Calculations of $\delta_C$ to $10\%$ are required...
\( \delta_C \) corrects the Fermi matrix element for charge-dependent forces:

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Calculations of \( \delta_C \) to 10% are required... *can we trust them??*
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**Approaches:** ↝ The shell model – the standard; Woods-Saxon vs. Hartree-Fock radial wavefunctions. Towner and Hardy *PRC* 82, 065501 (2010)
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$$|\Psi\rangle = |T \ T_3\rangle + \epsilon |T \pm 1 \ T_3\rangle$$

and nuclei have charge

$\Rightarrow$ separate effects into $\delta_{C_1}$ and $\delta_{C_2}$:

1) Configuration mixing

2) Radial overlap
Emphasis (scrutiny) now is on $\delta_C$

$\delta_C$ corrects the Fermi matrix element for charge-dependent forces:

$$M_F^2 = M_0^2 (1 + \delta_C)$$

Calculations of $\delta_C$ to **10%** are required... *can we trust them??*

**Approaches:**

⇝ The shell model – the standard; Woods-Saxon vs. Hartree-Fock radial wavefunctions. Towner and Hardy PRC **82**, 065501 (2010)


“...radial excitations are neglected by TH, and as a result, the transition operator violates the isospin commutation relations.”

“...radial excitations... are estimated to decrease the ISB corrections...”
Emphasis (scrutiny) now is on $\delta_C$

$\delta_C$ corrects the Fermi matrix element for charge-dependent forces:

$$M_F^2 = M_0^2 (1 + \delta_C)$$

Calculations of $\delta_C$ to 10% are required... *can we trust them??*

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⇝ The shell model – the standard; Woods-Saxon vs. Hartree-Fock radial wavefunctions. Towner and Hardy *PRC* 82, 065501 (2010)

⇝ Miller and Schwenk – critique in *PRC* 78, 035501 (2008) and *PRC* 80, 064319 (2009)


How well does a model’s calculated $\delta_C$ align the $ft$ values?

Define:

$$\delta_{C}^{\text{exp}} \equiv 1 + \delta_{NS} - \frac{A}{ft(1 + \delta'_{R})}$$

and fit to the model-calculated $\delta_C$’s, with $A$ a free parameter of the fit.

$(A$ corresponds to $\langle Ft\rangle$...if CVC holds!)
Testing calculations of ISB effects

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![Graph](image-url)
Testing calculations of ISB effects

How well does a model’s calculated $\delta_C$ align the $ft$ values?

Shell model with Woods-Saxon radial wavefunctions (TH):

best-fit $\langle Ft \rangle = 3072.0 \pm 0.6$ s

$\chi^2/13 = 0.4$, CL = 96%
Testing calculations of ISB effects

How well does a model’s calculated $\delta_C$ align the $ft$ values?

Shell model with Hartree-Fock radial wavefunctions (TH):

(would get $\langle Ft \rangle = 3071.3 \pm 0.8 \text{ s}$)

$\chi^2/13 = 2.1$, CL = 1%
How well does a model’s calculated $\delta_C$ align the $ft$ values?

Isovector monopole resonance (Auerbach):

(would get $\langle \mathcal{F}t \rangle = 3087.7 \pm 1.9$ s)

$\chi^2/13 = 11.2, \quad$ CL = 0%
Testing calculations of ISB effects

How well does a model’s calculated $\delta_C$ align the $ft$ values?

Relativistic RPA with PK01 eff. interaction (Liang):

$\langle F t \rangle = 3078.9 \pm 1.0 \text{ s}$

$\chi^2/8 = 3.1 \quad \text{CL} = 0\%$

Z of daughter
Testing calculations of ISB effects

How well does a model’s calculated $\delta_C$ align the $ft$ values?

Relativistic RPA with DD-ME2 eff. interaction (Liang):

(would get $\langle F_t \rangle = 3081.4 \pm 1.1$ s)

$\chi^2/8 = 2.4, \text{ CL } = 2\%$
Testing calculations of ISB effects

How well does a model’s calculated $\delta_C$ align the $ft$ values?

Density functional theory (Satuła):

(would get $\langle Ft \rangle = 3070.0 \pm 1.4$ s)

$\chi^2/12 = 5.2$, CL = 0%
Testing calculations of ISB effects

How well does a model’s calculated $\delta_C$ align the $ft$ values?

Density functional theory (Satuła):

(would get $\langle F_t \rangle = 3069.1 \pm 0.8$ s)

$\chi^2/11 = 1.5$, CL = 15%
(if exclude $^{62}$Ga)
How else can we test ISB corrections?

Using the 13 most precisely measured $T = 1$ cases to date lends strong support to the shell model approach with Woods-Saxon radial wavefunctions
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$$
\langle \delta_C - \delta_{NS} \rangle = 0.57\%
$$
How else can we test ISB corrections?

Using the 13 most precisely measured $T = 1$ cases to date lends strong support to the shell model approach with Woods-Saxon radial wavefunctions.

Another good test: measure it in different case(s) where it is expected to be larger.

\[ \delta_C - \delta_{NS} = 2.3(4)\% \]

B.A. Brown
$^{32}\text{Ar}: a T = 2$ superallowed decay
$^{32}$Ar: a $T = 2$ superallowed decay

W-S vs. H-F: $\delta_C$ calcs seem to depend on $T$ . . .

$\beta - \nu$ correlations

new cases for $V_{ud}$ . . .?
Experiment at NSCL

Clover 1 (G1)

Clover 2 (G2)

Clover 3 (G3)

Al foil

32Ar

31S + p

0+; 2

5046

2212

1169

461

90

55%

23%

21%

0.2%

1.5%

0.1%

1+; 1

1+; 1

1+; 1

32Cl

32Ar

120% (G5)

80% (G4)

1.5%

0.1%

0.2%

5046

2212

1169

461

90

55%

23%

21%

1+; 1

1+; 1

1+; 1
$^{32}$Ar proton spectra

The diagram shows the nuclear reactions and transitions involving $^{32}$Ar along with proton spectra. The ISOLDE experiment is highlighted, indicating the analysis of superallowed transitions in $^{32}$Cl.
$^{32}$Ar proton spectra

\[ \text{Counts / 0.5 keV} \]

\[ N_{\text{det}} - N_{\text{WC}} \]

\[ E_p \text{ [keV]} \]

\[ \sigma \]

\[ 10^4 \]

\[ 10^3 \]

\[ 10^2 \]

\[ 10^1 \]

\[ 1 \]

\[ 10^{-1} \]

\[ 10^{-2} \]

\[ 10^{-3} \]

\[ 10^{-4} \]

\[ 4 \]

\[ 2 \]

\[ -2 \]

\[ -4 \]

\[ 2000 \]

\[ 2500 \]

\[ 3000 \]

\[ 3500 \]

NSCL

ISOLDE
$^{32}\text{Ar}$ proton spectra

Dan Melconian

5/31/2012 – 12
**Branching ratios**

\[ N_p / N_{\text{Ar}} = 20.79(14)\% \]

Summary of systematic uncertainties on the absolute superallowed branch in \(^{32}\text{Ar}\) decay.

<table>
<thead>
<tr>
<th>Component</th>
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Branching ratios

protons:
\( \frac{N_p}{N_{Ar}} = 20.79(14)\% \)

gammas:
\( \frac{N_\gamma}{N_{Ar}} = 1.92(9)\% \)

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**Result from $^{32}\text{Ar}$**

**Branching ratios**

protons:

$N_p/N_{^{32}\text{Ar}} = 20.79(14)\%$

gammas:

$N_\gamma/N_{^{32}\text{Ar}} = 1.92(9)\%$

$f = 3505(8)$ and $t_{1/2} = 100.5(3)$ ms

$\delta_{NS} = -0.26(2)\%$

$\delta'_R = 1.41(3)$

$\Rightarrow f t = 1538(14) \text{ s}$

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\[ \Rightarrow ft = 1538(14) \text{ s} \]

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\[ \delta_{NS}^{\text{exp}} \equiv 1 + \delta_{NS} - \frac{3072.1(8) \text{ s}}{ft(1 + \delta'_R)} \]

Re-arrange \(\mathcal{F}t = ft(1 + \delta'_R)(1 + \delta_{NS} - \delta_C)\) and take \(\langle \mathcal{F}t \rangle\) from \(T = 1\) cases:
**Result from $^{32}$Ar**

**Branching ratios**

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  \[ N_p / N_{^{32}Ar} = 20.79(14)\% \]

- **gammas:**
  \[ N_\gamma / N_{^{32}Ar} = 1.92(9)\% \]

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<td>Implanted $^{32}$Ar’s</td>
<td>±0.23</td>
</tr>
<tr>
<td>$p_0$ branch</td>
<td>±0.52</td>
</tr>
<tr>
<td>$p_1$ branch</td>
<td>±0.04</td>
</tr>
</tbody>
</table>

Re-arrange \[ \mathcal{F}t = ft(1 + \delta'_R)(1 + \delta_{NS} - \delta_C) \]
and take \[ \langle \mathcal{F}t \rangle \text{ from } T = 1 \text{ cases:} \]

\[ \delta^{\text{exp}}_C \equiv 1 + \delta_{NS} - \frac{3072.1(8) \text{ s}}{ft(1 + \delta'_R)} \]

- **experimental value:** \[ \delta^{\text{exp}}_C = 2.1(8)\% \]
- **versus predicted:** \[ \delta^{\text{calc}}_C = 2.0(4)\% \]

**1st (semi-)precise measurement of a $T = 2$ pure Fermi decay**

Bhattacharya *et al.*, PRC 77 065503 (2008)
Bhattacharya et al., PRC 77, 065503 (2008) – measured SA branch
Wrede et al., PRC 81, 055503 (2010) – improved $^{32}$Cl mass
Signoracci and Brown, PRC 84, 031301(R) (2011) – revised calc of $\delta_C$

$\delta_{C1} = 0.60 \rightarrow 0.43(20)\%$ and $\delta_{C2} = 1.40(40)$

$\delta_C = 1.8(8)\%$ (expt) vs. $1.8(5)$ (theory)

$\gamma$ branch $\rightarrow$ about half of the total systematic
Motivation for measuring $^{32}\text{Cl}$ branches

1. **Original motivation**: $^{32}\text{Cl}$ is a daughter of the $\beta$-delayed proton decay of $^{32}\text{Ar} \approx 60\%$ of the time:

\[
\begin{align*}
0^+, T_Z &= 2 \\
32\text{Ar} &\rightarrow 0^+, T_Z = 2 \quad 5046 \\
32\text{Cl} &\rightarrow 1^+, 0.1\% \quad 2212 \\
&\rightarrow 1^+, 0.2\% \quad 1169 \\
&\rightarrow 1^+, 15\% \quad 1169 \\
&\rightarrow 1^+, 0.2\% \quad 1169 \\
&\rightarrow 0^+, 21\% \quad 3079 \\
&\rightarrow 5/2^+, 23\% \quad 2236 \\
&\rightarrow 3/2^+ \quad 1249 \\
&\rightarrow 1/2^+ 0 \quad 0
\end{align*}
\]

$^{31}\text{S} + p \rightarrow ^{32}\text{Cl}$

$^{32}\text{S}$

$^{32}\text{Ar}$

$\Rightarrow$ precise knowledge of $^{32}\text{Cl} \gamma$ branches provides an *in situ* calibration for $^{32}\text{Ar}$
Motivation for measuring $^{32}$Cl branches

1. Original motivation: $^{32}$Cl is a daughter of the $\beta$-delayed proton decay of $^{32}$Ar $\approx 60\%$ of the time: $\Rightarrow$ in situ calibration of HPGe

2. While analyzing: Found isospin-symmetry-breaking effects in $^{32}$Cl’s isobaric analogue decay is predicted to be very large!
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\[ \Delta E = 188.2 \pm 1.2 \text{ keV} \]

$\Rightarrow \quad \delta_{C1} \propto \frac{1}{(\Delta E)^2}$ is big
Motivation for measuring $^{32}\text{Cl}$ branches

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---

![Graph showing calculated and measured values of $\delta_C - \delta_{NS}$ for various isotopes, with a note to measure it and test model calculations.]
Observed $\gamma$ spectrum in HPGe

- MARS beamline
- $^{32}\text{Cl}$
- aluminum degraders
- aluminized Mylar tape
- BC404 scintillator
- shielding
- 70% HPGe

1 cycle

0 0.80 0.98 1.98 2.78 2.96

T [sec]
Aside from \( \sim 7\% \) \(^{30}\text{S}\) contamination, we have an essentially background-free spectrum.
Efficiency of the HPGe

Absolutely **critical** to the success of the experiment!

\[
\begin{align*}
50 - 1400 \text{ keV} : & \pm 0.2\% \\
1.4 - 3.5 \text{ MeV} : & \pm 0.4\%
\end{align*}
\]

- Helmer *et al.*, NIM **A511**, 360 (2003), and
Efficiency of the HPGe

Absolutely critical to the success of the experiment!

50 – 1400 keV : ± 0.2% \hspace{1cm} \text{Hardy et al., Appl. Radiat. Isot. 56, 65 (2002),}\hspace{1cm} \text{Helmer et al., NIM A511, 360 (2003), and}\hspace{1cm} \text{Helmer et al., Appl. Radiat. Isot. 60, 173 (2004)}

1.4 – 3.5 MeV : ± 0.4% \hspace{1cm} \text{PENELOEPE MC simulation}\hspace{1cm} \text{Sempau et al., PLB 71, 307 (1977)}

3.5 – 5 MeV : ± 1%

5 MeV and above : ± 5%
Skipping details of the analysis...

Starting from Détraz et al., Nucl. Phys. A203, 414 (1973)...
Starting from Détraz et al., Nucl. Phys. **A203**, 414 (1973)…

- Use known ground state branch (Armini et al., Phys. Rev. **165** (1967))
Skipping details of the analysis.


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Fit the $\beta$ and $\gamma$ branches to reproduce observed yields
Deduced branch to the IAS

- considered 51 excited states in $^{32}$S
- found 3 new $\beta$ transitions and put limits on 2 others
- found 22 new $\gamma$ transitions and put limits on 10 others
- improved precision of known branches by $\approx 1$ order of magnitude
- USD, USDA and USDB shell-model calculations
  $\Rightarrow$ 30 states with $E_x > 7.8$ MeV sum to 0.60(10)% $\beta$ strength
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  $\Rightarrow$ 30 states with $E_x > 7.8$ MeV sum to $0.60(10)\%$ $\beta$ strength

$\Rightarrow$ branch to IAS found to be $\boxed{(22.47 \pm 0.13^{+0.16}_{-0.12})\%}$

$\pm 0.10\%$ from HPGe photopoint efficiency
$\pm 0.11\%$ from ground state branch

- cuts made on data ($\beta$/HI ratio, $\beta$-$\gamma$ timing, start/stop of counting times)
- total $\gamma$ efficiency (summing)
- ENSDF uncertainties; which effective interaction
- efficiency of plastic scintillator

Melconian et al., PRC 85, 025501 (2012)
This $1^+ \rightarrow 1^+$ decay is mixed Fermi/Gamow-Teller... 😞

So *how do we learn anything* (without measuring a correlation)?
This $1^+ \rightarrow 1^+$ decay is **mixed** Fermi/Gamow-Teller… 😐

So *how do we learn anything* (without measuring a correlation)?

Make assumptions about Gamow-Teller contributions and deduce $M_F$

Fortunately, the shell-model predicts a very **weak** contribution:

$$M_{GT} = 0.012 \text{ (USD)}, \ 0.065 \text{ (USDA)} \text{ and } 0.036 \text{ (USDB)},$$

or $\langle M_{GT} \rangle = 0.04(3)$  (vs. $M_F = \sqrt{2}$) 😁
This $1^+ \rightarrow 1^+$ decay is **mixed** Fermi/Gamow-Teller… 😞

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or

$$\langle M_{GT} \rangle = 0.04(3) \quad (\text{vs.} \quad M_F = \sqrt{2})$$

$$\Rightarrow$$

Gamow-Teller is a small correction to an otherwise **pure Fermi transition**!

$$\left( \frac{M_{GT}^2}{M_F^2} < 0.1\% \right)$$
\( ft \) value of \(^{32}\text{Cl} \) decay to IAS

\[
\left( \text{phase space} \right) \left( \text{partial half-life} \right) \equiv ft \quad \text{is the "comparative half-life"}
\]

1. Phase space factor:

\( \Rightarrow \) Shi et al., PRA 72, 022510 (2005)

\( \Rightarrow \) Wrede et al., PRC 81, 055503 (2010)

\( \Rightarrow \) Kankainen et al., PRC 82, 052501 (2010)

\( \Rightarrow \) Audi et al., NPA 729, 337 (2003)

\( \Rightarrow \) Towner shell model for \( C(E) \)

\[
f = \int F(Z, E) C(E) pE(E - E_0)^2 dE
\]

\( \Rightarrow \quad f = 2411.6 \pm 2.4 \)
\( ft \) value of \(^{32}\text{Cl}\) decay to IAS

\[
(\text{phase space })(\text{partial half-life}) \equiv ft \quad \text{is the “comparative half-life”}
\]

1. Phase space factor: \( f = 2411.6 \pm 2.4 \)

2. Partial half-life:

\( \Rightarrow \) Armini et al., PR 165, 1194 (1968): \( t_{1/2} = 298(1) \) ms

\( \Rightarrow \) IAS branch (this work): \( \text{Br} = (22.47^{+0.21}_{-0.18})\% \)

\( \Rightarrow \) Small electron-capture fraction:

\[
P_{EC} = 0.071\% 
\]

\[
t = \frac{t_{1/2}}{\text{Br}} (1 + P_{EC})
\]

\( \Rightarrow \quad t = 1.327(13) \) s
$ft$ value of $^{32}\text{Cl}$ decay to IAS

$\left( \text{phase space} \right) \left( \text{partial half-life} \right) \equiv ft$ is the “comparative half-life”

1. Phase space factor: $f = 2411.6 \pm 2.4$
2. Partial half-life: $t = 1.327 \pm 0.013 \text{ s}$

\[
\begin{cases} 
ft = 3200(30) \text{ s} 
\end{cases}
\]
\( f t \) value of \(^{32}\text{Cl}\) decay to IAS

\[
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\[ f t = 3200(30) \text{ s} \]

And again interpret result as a measure of isospin-mixing:

\[
\delta_{\text{C}}^{\text{exp}} \equiv 1 + \delta_{\text{NS}} - \frac{\langle F_t \rangle}{ft(1 + \delta'_R)}
\]
$ft$ value of $^{32}$Cl decay to IAS

\[
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\]

1. Phase space factor: \( f = 2411.6 \pm 2.4 \)
2. Partial half-life: \( t = 1.327 \pm 0.013 \text{ s} \)

\[
\begin{align*}
ft & = 3200(30) \text{ s} \\
\delta_C^{\text{exp}} & \equiv 1 + \delta_{NS} - \frac{3071.8(8) \text{ s}}{ft(1 + \delta_R^{'})} \\
& -0.15(2)\% \\
& 1.42(3)\%
\end{align*}
\]

And again interpret result as a measure of isospin-mixing:
\[ \left( \text{phase space} \right) \left( \text{partial half-life} \right) \equiv ft \text{ is the "comparative half-life"} \]

1. Phase space factor: \( f = 2411.6 \pm 2.4 \)
2. Partial half-life: \( t = 1.327 \pm 0.013 \) s

And again interpret result as a measure of isospin-mixing:

\[
\delta^{\text{exp}}_C \equiv 1 + \delta_{NS} - \frac{\langle ft \rangle}{ft(1 + \delta'_R)} = 5.3(9)\%
\]

Melconian et al., PRL 107, 182301 (2011)

Largest-ever calculated ISB effect in a superallowed decay
Comparison of expt and theory

\[ (\Delta E_{\text{theor}} / \Delta E_{\text{expt}})^2 \text{ scaling} \Rightarrow \]

\[ \delta_{C1} = 3.75(45)\% \]

reproduce charge radius of \(^{31}\text{S}\) \Rightarrow

\[ \delta_{C2} = 0.85(3)\% \]
Comparison of expt and theory

\[
(\Delta E_{\text{theor}}/\Delta E_{\text{expt}})^2 \text{ scaling} \Rightarrow \delta_{C1} = 3.75(45)\% \\
\text{reproduce charge radius of } ^{31}\text{S} \Rightarrow \delta_{C2} = 0.85(3)\% \\
\Rightarrow \delta_{C}^{\text{th}} = 4.6(5)\% \text{ agrees with } \delta_{C}^{\exp} = 5.3(9)\% \\
\text{another agreement with the shell model approach}
\]
Mass-32 results vs. other models?

TH approach agrees with all tests of ISB in superallowed decays
Mass-32 results vs. other models?

TH approach agrees with all tests of ISB in superallowed decays

Miller and Schwenk still not convinced

“You say/imply that observing a large ISB effect provides a strong test of theories. I think it is easier to calculate a large effect than a small one.”

G. Miller
Mass-32 results vs. other models?

TH approach agrees with all tests of ISB in superallowed decays

What about other approaches...?

Miller and Schwenk still not convinced (large = easy; TH formalism inherently flawed)
TH approach agrees with all tests of ISB in superallowed decays

What about other approaches...?

⇝ Miller and Schwenk still not convinced (large = easy; TH formalism inherently flawed)
⇝ Auerbach intrigued

“there is no question of collectivity here and the detailed shell-model calculation might be valid, explaining the $^{32}$Cl large $\delta_{C_1}$ number.”
TH approach agrees with all tests of ISB in superallowed decays

What about other approaches...?

Miller and Schwenk still not convinced (large = easy; TH formalism inherently flawed)
Auerbach intrigued (IVMR more applicable to $\delta_C^2$)
Satuła et al. still working (testing new functionals)

“We have also a new functional fitted to $N \approx Z$ nuclei of interest...[that] is perfect to investigate sensitivity of the calculated $\delta_C$ with respect to parametrization of the nuclear EDF.”
TH approach agrees with **all** tests of ISB in superallowed decays

What about other approaches...?

⇝ Miller and Schwenk still not convinced (large = easy; TH formalism inherently flawed)
⇝ Auerbach intrigued (IVMR more applicable to $\delta_{C2}$)
⇝ Satuła et al. still working (testing new functionals)
⇝ Liang et al. also still working (including pairing and deformation)

“At this moment, we are developing codes to explore the pairing and deformation effects on the isospin-mixing. Definitely the cases of $T = 2$ transitions including $^{32}$Ar will also be investigated.”
Mass-32 results vs. other models?

TH approach agrees with all tests of ISB in superallowed decays

What about other approaches…?

⇝ Miller and Schwenk still not convinced (large = easy; TH formalism inherently flawed)

⇝ Auerbach intrigued (IVMR more applicable to $\delta_{C2}$)

⇝ Satuła et al. still working (testing new functionals)

⇝ Liang et al. also still working (including pairing and deformation)

We all look forward to development of all these theoretical approaches, as well as to improved experimental precision in the mass-32 (and other) cases!
Summary

- $V_{ud}$ and CKM unitarity constrain extensions to the SM
- Isospin-symmetry-breaking corrections are important and now under scrutiny
- Tests of ISB effects:
  1. using existing 13 cases, where $\langle \delta_C - \delta_{NS} \rangle = 0.6\%$
  2. BIG in $^{32}\text{Ar}$ (1.8%) 
  3. HUGE in $^{32}\text{Cl}$ (5.3%) 
     (“bread-and-butter” experiments can lead to surprising and exciting results!)
- Shell model matches well with experiment in all cases
- Other groups are beginning to develop complementary models of ISB in superallowed decays – will be an important check and may lead to better systematics on $V_{ud}$
Collaborators:

$^{32}$Ar: A. García, S. Triambak, M. Bhattacharya, A. Komives, E.G. Adelberger, B.A. Brown, T. Glasmacher, P.F. Mantica, H.E. Swanson, ... (UW, NSCL, Notre Dame, DePauw, FSU, ...)

$^{32}$Cl: A. García, I.S. Towner, J.C. Hardy, S. Triambak, V.E. Iacob, L. Trache, R.E. Tribble (UW, TAMU)

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⇝ The organizers for the invitation and you for your attention

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