Systematic $^3$He Potential for 1p-shell nuclei

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Elastic Scattering

Incoming Plane Wave
\[ \varphi(r) = Ae^{i\vec{k} \cdot \vec{r}} \]

Outgoing Plane and Spherical Wave
\[ \varphi(r) = Ae^{i\vec{k} \cdot \vec{r}} + f(\theta, \varphi)Ae^{\frac{(i\vec{k} \cdot \vec{r} - d)}{r_d}} \]

Possible Interactions
- Incoming plane wave passes through the nucleus without interaction (\(k=\hat{k}\))
- Incoming plane wave releases energy to excite nucleus and but does not react (\(k \neq \hat{k}\))
- Incoming plane wave reacts with nucleus and a new particle is emitted (\(k \neq \hat{k}\))
Nuclear Optical Model Potential

- Simplifies many-body problems into two-body problems for collisions of nuclei
- Models the system with a complex potential to accommodate elastic and inelastic scattering
- Solve the Schrodinger Equation to predict reaction cross sections

\[ \varepsilon \varphi = (-\frac{h^2}{2m_1} \nabla^2 - \frac{h^2}{2m_2} \nabla^2 + \Phi(r))\varphi \]

(1=incident particle, 2= target nucleus)

- This optical model potential is necessary for evaluating reaction rates
Why study Optical Potentials?

- $^{6}\text{Li}(^{3}\text{He},d)^{7}\text{Be}$: measured by Catania, Italy (Astrophysical)
- $^{14}\text{O}(d,^{3}\text{He})^{13}\text{N}$, and $^{14}\text{O}(d,t)^{13}\text{O}$, measured at Saclay, France (Nuclear Structure)
- Need an optical potential to analyze these reactions but previous models have difficulty fitting light targets
- Focus on fitting of 1-p shell ($A_T \sim 6$ to 18)
- This potential is usable in many different light target experiments
Finding the Global Optical Potential

Phenomenological

- Fit data from many cross sections
- Each angular distribution has many possible potentials, but through analysis of many cross sections, a general potential $U(r)$ can be estimated.

Microscopic

- Uses the double folding model
- Fold the nucleon-nucleon interactions with nucleon density distributions
Form of the Global Potential

\[ U(r) = -V_r f_{ws}(r,R_o,a_o) - iW_v f_{ws}(r,R_w,a_w) \]

\[ -iW_s (-4a_w) \frac{d}{dr} f_{ws}(r,R_w,a_w) + V_c \]

Woods-Saxon form factor

\[ f_{ws}(r,R,a) = \frac{1}{1 + \exp(r - R)/a} \]

Coulomb Potential

\[ V_c(r) \]

\[ \begin{cases} 
\frac{Z_p Z_t e^2}{r} & (r \geq R_c) \\
\frac{Z_p Z_t e^2}{2R_c} (3 - \frac{r^2}{R_c^2}) & (r \geq R_c)
\end{cases} \]

\[ R_c = r_c A_T^{1/3} + r_c^{(0)} \]

R_c = Coulomb Radius

\[ r_c = 1.24 \text{fm} \]

Real Volume Potential

\[-V_r f_{ws}(r, R_0, a_0)\]

Parameterizing the Real Volume Potential

Energy Dependence

\[ V_r = V_0 + V_e(E - E_C) \]

\[ E_C = \alpha \frac{6Z_pZ_t e^2}{5R_c} \]

Mass Dependence

\[ R_c = r_c A_T^{1/3} + r_c^{(0)} \]

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Imaginary Volume Potential

\[-iW_v f_{ws}(r,R_w,a_w)\]

Parameterizing the imaginary volume potential

**Fermi-type Energy Dependence**

\[ W_s = \frac{W_{s0} + W_{st}}{1 + \exp \left( \frac{(E - E_c) - W_{se0}}{W_{se0}} \right) } \]

\[ E_c = \alpha \frac{6Z_pZ_te^2}{5R_c} \]

**Mass Dependence**

\[ R_W = r_wA_T^{1/3} + r_w^{(0)} \]

Imaginary Surface Potential

\[-iW_s(-4a_w)\frac{d}{dr} f_{ws}(r,R_w,a_w) +\]

Parameterizing the imaginary surface potential

Fermi-type Energy Dependence

\[W_v = \frac{W_{v0}}{1 + \exp\left(\frac{W_{ve0} - (E - E_c)}{W_{sew}}\right)}\]

\[E_c = \alpha \frac{6Z_pZ_tE^2}{5R_c}\]

Mass Dependence

\[R_W = r_w A_T^{1/3} + r_w^{(0)}\]

Database

- Analysis of 53 cross sections
- 19 in $^6$Li fitting, 34 in A~12 fitting.
MINOPT

- Allows simultaneous fitting of parameters

OPTICS

- Solves the Schrodinger equation for a given potential
- Provide the theoretical cross sections.

MINUIT

- the statistical optimization program
- varies parameters to search for a minimization of chi-squares

\[
\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}
\]

Tasks

- Eliminate results with nonphysical results
- Evaluate whether variable are confined
- Ensure stability of results
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<th>Error</th>
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Result – $^6\text{Li}$ Fitting
Results; A~12 Fitting

Boron - 10; 8-46.1 MeV

Carbon - 12; 25.3-44 MeV

Carbon - 13; 33.0-41 MeV

Carbon - 14; 37.9-72 MeV
Future Work

- Unite potentials in a single model
Acknowledgements

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