Last lecture

- Introduction to Digital Electronics
  - Widespread “binary” electronics
    - Have grown exponentially for decades
  - In everything!
  - Reproducible and resistant to noise
  - Fast and easy to use
- Process as binary levels
  - High or Low
  - Gates (AND, OR, NOT, etc.) to control logic flow
- Gates are just arrangements of switches
  - Mostly MOSFETs
  - 74XXX series are ICs with many gates
    - can be combined to control processes, arithmetic, etc.
- Can be bunched into PLDs, FPGAs, and ASICs
- Software become critical for application when numbers get large!
  - Assembly language and machine code
Digital to Analog Conversion

• DAC
  – Converts digital signal into an analog signal
  – Very broad range of uses
  – Interfaces analog and digital electronics
• Computer control of any(every)thing
• Some digital ICs can directly control some devices
  – LEDs
  – Low power, low current, on-off
Purpose

• To convert digital values to analog voltages
• Performs inverse operation of the Analog-to-Digital Converter (ADC)
• $V_{OUT} \propto \text{Digital Value}$
DACs

• Types
  – Binary Weighted Resistor
  – R-2R Ladder
  – Multiplier DAC
    • The reference voltage is constant and is set by the manufacturer.
  – Non-Multiplier DAC
    • The reference voltage can be changed during operation.

• Characteristics
  – Comprised of switches, op-amps, and resistors
  – Provides resistance inversely proportion to significance of bit
Binary Weighted Resistor

\[ R_f = R \]

\[ \sum I_i \]

- \(-V_{REF}\)

MSB

LSB

\( V_o \)
Binary Representation

Least Significant Bit

Most Significant Bit

\[ \sum I_i \]

\[ R_f = R \]

\[ V_o \]

\[ -V_{REF} \]
Binary Representation

Most Significant Bit

- $V_{REF}$

SET

CLEARED

Least Significant Bit

\[
(\ 1 \ 1 \ 1 \ 1 )_2 = (15)_{10}
\]
Binary Weighted Resistor

- “Weighted Resistors” based on bit
- Reduces current by a factor of 2 for each bit
Binary Weighted Resistor

• Result:

\[ \sum I = V_{REF} \left( \frac{B_3}{R} + \frac{B_2}{2R} + \frac{B_1}{4R} + \frac{B_0}{8R} \right) \]

\[ V_{OUT} = I \cdot R_f = V_{REF} \left( B_3 + \frac{B_2}{2} + \frac{B_1}{4} + \frac{B_0}{8} \right) \]

– \( B_i = \text{Value of Bit } i \)
Binary Weighted Resistor

• More Generally:

\[ V_{OUT} = V_{REF} \sum \frac{B_i}{2^{n-i-1}} \]

\[ = V_{REF} \cdot \text{Digital Value} \cdot \text{Resolution} \]

– \( B_i \) = Value of Bit \( i \)
– \( n \) = Number of Bits
R-2R Ladder
R-2R Ladder

- Same input switch setup as Binary Weighted Resistor DAC
- All bits pass through resistance of 2R
R-2R Ladder

- The less significant the bit, the more resistors the signal must pass through before reaching the op-amp
- The current is divided by a factor of 2 at each node
R-2R Ladder

- The current is divided by a factor of 2 at each node
- Analysis for current from \((001)_{2}\) shown below

\[ I_0 = - \frac{V_{REF}}{2R + 2R \parallel 2R} = \frac{V_{REF}}{3R} \]
R-2R Ladder

• Result:

\[ I = \frac{V_{REF}}{3R} \left( \frac{B_2}{2} + \frac{B_1}{4} + \frac{B_0}{8} \right) \]

\[ V_{OUT} = \frac{R_f}{R} V_{REF} \left( \frac{B_2}{2} + \frac{B_1}{4} + \frac{B_0}{8} \right) \]

– \( B_i \) = Value of Bit \( i \)
R-2R Ladder

• If $R_f = 6R$, $V_{OUT}$ is same as Binary Weighted:

$$I = \frac{V_{REF}}{3R} \sum \frac{B_i}{2^{n-i}}$$

$$V_{OUT} = V_{REF} \sum \frac{B_i}{2^{n-i-1}}$$

– $B_i$ = Value of Bit $i$
R-2R Ladder

- Example:
  - Input = (101)_2
  - $V_{\text{REF}} = 10 \text{ V}$
  - $R = 2 \Omega$
  - $R_f = 2R$

\[
I_0 = \frac{-V_{\text{REF}}}{2R + 2R||2R} = \frac{V_{\text{REF}}}{3R} = -1.67 \text{ mA}
\]

\[
I_{\text{op-amp}} = \frac{I_0}{8} + \frac{I_0}{2} = -1.04 \text{ mA}
\]

\[
V_{\text{OUT}} = -I_{\text{op-amp}} R_f = 4.17 \text{ V}
\]
## Pros & Cons

<table>
<thead>
<tr>
<th>Pros Weighted</th>
<th>R-2R</th>
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<tbody>
<tr>
<td><strong>Pros</strong></td>
<td></td>
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<tr>
<td>Easily understood</td>
<td>Only 2 resistor values</td>
</tr>
<tr>
<td>Limited to ~ 8 bits</td>
<td>Easier implementation</td>
</tr>
<tr>
<td>Large # of resistors</td>
<td>Easier to manufacture</td>
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<tr>
<td>Susceptible to noise</td>
<td>Faster response time</td>
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<tr>
<td>Expensive</td>
<td></td>
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<tr>
<td>Greater Error</td>
<td>More confusing analysis</td>
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<table>
<thead>
<tr>
<th>Cons</th>
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DAC characteristics

- Resolution
- Reference Voltages
- Settling Time
- Linearity
- Speed
- Errors
Resolution

• **Resolution**: is the amount of variance in output voltage for every change of the LSB in the digital input.

• How closely can we approximate the desired output signal (Higher Res. = finer detail = smaller Voltage divisions)

• A common DAC has a 8 - 12 bit Resolution

\[
\text{Resolution} = V_{LSB} = \frac{V_{\text{Ref}}}{2^N}
\]

N = Number of bits
Resolution

Poor Resolution (1 bit)

Vout

Desired Analog signal

Approximate output

Digital Input

Better Resolution (3 bit)

Vout

Desired Analog signal

Approximate output

Digital Input
Reference Voltage

• **Reference Voltage**: A specified voltage used to determine how each digital input will be assigned to each voltage division.

• Types:
  – **Non-multiplier**: internal, fixed, and defined by manufacturer
  – **Multiplier**: external, variable, user specified
Reference Voltage

Non-Multiplier: \( V_{\text{ref}} = C \)

Multiplier: \( V_{\text{ref}} = A \sin(\omega t) \)

Assume 2 bit DAC
Settling Time

- **Settling Time:** The time required for the input signal voltage to settle to the expected output voltage (within +/- $V_{\text{LSB}}$).

- Any change in the input state will not be reflected in the output state immediately. There is a time lag, between the two events.
Settling Time

Analog Output Voltage

Expected Voltage

\(+V_{\text{LSB}}\)

\(-V_{\text{LSB}}\)

Settling time
Linearity

• **Linearity**: is the difference between the desired analog output and the actual output over the full range of expected values.

• Ideally, a DAC should produce a linear relationship between a digital input and the analog output, this is not always the case.
Linearity

Linearity (Ideal Case)

Perfect Agreement

NON-Linearity (Real World)

Miss-alignment
Speed

• **Speed**: Rate of conversion of a single digital input to its analog equivalent

• **Conversion Rate**
  – Depends on clock speed of input signal
  – Depends on settling time of converter
Errors

• Non-linearity
  – Differential
  – Integral
• Gain
• Offset
• Non-monotonicity
Errors

- **Differential Non-Linearity**: Difference in voltage step size from the previous DAC output (Ideally All DLN’s = 1 $V_{LSB}$)
• **Integral Non-Linearity:** Deviation of the actual DAC output from the ideal (Ideally all INL’s = 0)
**Errors**

- **Gain Error**: Difference in slope of the ideal curve and the actual DAC output

  - **High Gain Error**: Actual slope greater than ideal
  - **Low Gain Error**: Actual slope less than ideal
Errors

• **Offset Error:** A constant voltage difference between the ideal DAC output and the actual.
  – The voltage axis intercept of the DAC output curve is different than the ideal.
Errors

- **Non-Monotonic**: A decrease in output voltage with an increase in the digital input.
Applications

- Generic use
- Circuit Components
- Digital Audio
- Function Generators/Oscilloscopes
- Motor Controllers
Generic

- Used when a continuous analog signal is required.
- Signal from DAC can be smoothed by a Low pass filter.
Circuit Components

• Voltage controlled Amplifier
  – digital input, External Reference Voltage as control

• Digitally operated attenuator
  – External Reference Voltage as input, digital control

• Programmable Filters
  – Digitally controlled cutoff frequencies
Digital Audio

• CD Players
• MP3 Players (!)
• Digital Telephone/Answering Machines
Function Generators

- Digital Oscilloscopes
  - Digital Input
  - Analog Output

- Signal Generators
  - Sine wave generation
  - Square wave generation
  - Triangle wave generation
  - Random noise generation
Motion Control

- Cruise Control
- Valve Control
- Motor Control