Everything that can be invented has been invented.

attributed to Charles H. Duell, Commissioner, U.S. patent office, 1899, in letter to President McKinley.
1835 philosopher Auguste Comte considered the limits of human knowledge. In his book *Positive Philosophy*, he wrote that we would never understand the stars:

We see how we may determine their forms, their distances, their bulk, their motions, but we can never know anything of their chemical or mineralogical structure.

Auguste Comte (1798-1857)
Spectral Lines

Joseph von Fraunhofer
(1787-1826)
Spectral Lines

Joseph von Fraunhofer (1787-1826)

Identified “black” lines in Sunlight when dispersed by a prism. Regions with no emission. By 1814, Fraunhofer had cataloged over 475 of these lines.
Spectral Lines

Applications: Measure Elements and Velocities of Stars

• Fraunhofer identified one line in the Solar spectrum to correspond to a line of wavelength 590 nm observed when salt is burnt in a flame. Corresponds to Sodium in the Sun’s atmosphere.

• Studies of absorption lines in the Sun’s spectrum identified “Helium” (from Greek Helios for Sun) in 1868, not seen previously on Earth (and not discovered until 1895).

• Other studies of Sun showed that it contains a wide variety of elements.

• Other stars show similar (yet often different) absorption lines.
<table>
<thead>
<tr>
<th>Wavelength [nm]</th>
<th>Name</th>
<th>Atom</th>
</tr>
</thead>
<tbody>
<tr>
<td>393.368</td>
<td>K</td>
<td>Ca$^+$</td>
</tr>
<tr>
<td>396.849</td>
<td>H</td>
<td>Ca$^+$</td>
</tr>
<tr>
<td>410.175</td>
<td>h (Hδ)</td>
<td>H$^0$</td>
</tr>
<tr>
<td>422.674</td>
<td>g</td>
<td>Ca$^0$</td>
</tr>
<tr>
<td>434.048</td>
<td>G, Hγ</td>
<td>H$^0$</td>
</tr>
<tr>
<td>438.355</td>
<td>e</td>
<td>Fe$^0$</td>
</tr>
<tr>
<td>438.356</td>
<td>d</td>
<td>Fe$^0$</td>
</tr>
<tr>
<td>486.134</td>
<td>F (Hβ)</td>
<td>H$^0$</td>
</tr>
<tr>
<td>495.761</td>
<td>c</td>
<td>Fe$^0$</td>
</tr>
<tr>
<td>516.733</td>
<td>b4</td>
<td>Mg$^0$</td>
</tr>
<tr>
<td>517.270</td>
<td>b2</td>
<td>Mg$^0$</td>
</tr>
<tr>
<td>518.362</td>
<td>b1</td>
<td>Mg$^0$</td>
</tr>
<tr>
<td>527.039</td>
<td>E</td>
<td>Fe$^0$</td>
</tr>
<tr>
<td>588.997</td>
<td>D2</td>
<td>Na$^0$</td>
</tr>
<tr>
<td>589.594</td>
<td>D1</td>
<td>Na$^0$</td>
</tr>
<tr>
<td>627.661</td>
<td>a</td>
<td>O$_2$</td>
</tr>
<tr>
<td>656.281</td>
<td>C (Hα)</td>
<td>H$^0$</td>
</tr>
<tr>
<td>686.719</td>
<td>B</td>
<td>O$_2$</td>
</tr>
<tr>
<td>759.370</td>
<td>A</td>
<td>O$_2$</td>
</tr>
</tbody>
</table>
Spectral Lines

Stellar Classification

O6.5  B0  B6  A1  A5  F0  F5  G0  G5  K0  K5  M0  M5

Ca  H  H  Fe  Na  H
Spectral Lines
Spectral Lines

Application: Measure Elements and Velocities of Stars

Comparing Sun’s spectrum to other stars. For non-relativistic speeds, the line-of-sight (called the *recessional*) velocity is

\[
\frac{\lambda_{\text{obs}} - \lambda_{\text{rest}}}{\lambda_{\text{rest}}} = \frac{\Delta \lambda_{\text{obs}}}{\lambda_{\text{rest}}} = \frac{v_r}{c}
\]

In Vega, important Hydrogen line has \( \lambda_{\text{obs}} = 656.251 \text{ nm} \) compared to \( \lambda_{\text{rest}} = 656.281 \text{ nm} \), measured in the lab.

\[ v_r = c \frac{\lambda_{\text{obs}} - \lambda_{\text{rest}}}{\lambda_{\text{rest}}} = -13.9 \text{ km s}^{-1} \]

Vega also has a measured proper motion (perpendicular to line-of-sight) of \( \mu = 0.35077'' \text{ yr}^{-1} \). At \( r = 7.76 \text{ pc} \), the proper motion (or transverse velocity) is \( v_\theta = r \mu = 12.9 \text{ km s}^{-1} \). The total velocity is

\[ v = \sqrt{v_r^2 + v_\theta^2} = 19.0 \text{ km s}^{-1} \]
Quantum Mechanics I: Photo-electric Effect

Light (photons). Could have different brightness (# of photons per s) or photons of different frequencies

Electrons observed with Kinetic energy up to some measured maximum, $K_{\text{max}}$

Observations showed that $K_{\text{max}}$ does not depend on the brightness (intensity) of light.

But, observations showed that $K_{\text{max}}$ does depend on the frequency of the light.

Classically, amount of energy carried by light is $S = (1/\mu_0) |\mathbf{E} \times \mathbf{B}|$. No dependence on frequency.

Each metal has an observed cutoff frequency, such that electrons only released when $\nu > \nu_{\text{cut}}$. 
Quantum Mechanics I: Photo-electric Effect

Einstein knew of Planck’s work. He postulated that light stream contained particles (photons), each of which has some energy, \( E = h\nu = hc/\lambda \).

When photon hits metal, energy may be absorbed by an electron, which may become “unbound” from metal and released. The minimum binding energy of electrons in the metal is called its **Work function**, which gives the equation for \( K_{\text{max}} \):

\[
K_{\text{max}} = E - \phi = h\nu - \phi = hc/\lambda - \phi.
\]

For \( K_{\text{max}} = 0 \), \( \nu_{\text{cut}} = \phi/h \).

This is the cutoff frequency.

Albert Einstein (1879-1955). Won his Nobel Prize in 1921 for this work (he did not win one for his theories of relativity).
Quantum Mechanics II: Compton Scattering

Arthur Compton (1892-1962), shown here on the cover of Time Magazine in 1936

Compton studied experiments of collisions between X-rays (photons) and electrons. Collisions changed the frequency of the photons and scattered the electrons.

![Diagram of Compton Scattering](image)

Energy and Momentum conserved.

\[ \Delta \lambda = \lambda_f - \lambda_i = \left( \frac{h}{m_e c} \right)(1 - \cos \theta) \]

Proved that photons are massless, but carry momentum. Known as Compton Effect.

\( \lambda_C = \left( \frac{h}{m_e c} \right) \) is the Compton Wavelength
What does an atom look like?

In 19th century, people thought it looked something like this.
Rutherford Experiment

Old Atomic Model

Rutherford’s Atomic Model
Structure of Atoms

<table>
<thead>
<tr>
<th>Series Name</th>
<th>Symbol</th>
<th>Transition</th>
<th>Wavelength [nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balmer</td>
<td>Hα</td>
<td>3 to 2</td>
<td>656.3</td>
</tr>
<tr>
<td></td>
<td>Hβ</td>
<td>4 to 2</td>
<td>486.1</td>
</tr>
<tr>
<td></td>
<td>Hγ</td>
<td>5 to 2</td>
<td>434.0</td>
</tr>
<tr>
<td></td>
<td>Hδ</td>
<td>6 to 2</td>
<td>410.2</td>
</tr>
<tr>
<td></td>
<td>Hε</td>
<td>7 to 2</td>
<td>397.0</td>
</tr>
<tr>
<td></td>
<td>Hζ</td>
<td>8 to 2</td>
<td>388.9</td>
</tr>
<tr>
<td></td>
<td>Hη</td>
<td>9 to 2</td>
<td>383.5</td>
</tr>
<tr>
<td>H limit</td>
<td>∞ to 2</td>
<td></td>
<td>364.6</td>
</tr>
</tbody>
</table>

When dense Hydrogen gas is heated, it shows emission lines at exact wavelengths. By 1885, 14 spectral lines in Hydrogen had been measured. After much trial and error, Johann Balmer (1825-1898) found that the wavelengths of hydrogen corresponded to a pattern, now called the Balmer series made of the Balmer lines.

\[ \frac{1}{\lambda} = R_H \left( \frac{1}{4} - \frac{1}{n^2} \right), \ n=3, 4, 5, \ldots, \infty \]

\[ R_H = 1.09677583 \times 10^7 \text{ m}^{-1} \] is an experimentally measured quantity called the Rydberg constant for hydrogen.
Classical (incorrect) picture of atoms

As electron “accelerates” around nucleus, it emits radiation, loses energy and spirals into nucleus. The frequency of the radiation increases to infinity as R goes to zero. For hydrogen, the electron should collide with the nucleus in less than one-billionth of a second.
Structure of Atoms

Balmer realized that he could generalize this to:

$$\frac{1}{\lambda} = R_\text{H} \left( \frac{1}{m^2} - \frac{1}{n^2} \right), \quad m=2, \quad n=3, \ 4, \ 5, \ ..., \ \infty$$

And he predicted that other line series could be found with $m < n$ (both integers).

In 1906, Theodore Lyman confirmed the series for $m=1, \ n=2, \ 3, \ 4, \ ..., \$ as the Lyman Series.

In 1908, Friedrich Paschen confirmed the series for $m=3, \ n=4, \ 5, \ 6, \ ..., \$ as the Paschen Series.

<table>
<thead>
<tr>
<th>Series Name</th>
<th>Symbol</th>
<th>n to m</th>
<th>Wavelength [nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lyman</td>
<td>Lyα</td>
<td>2 to 1</td>
<td>121.567</td>
</tr>
<tr>
<td></td>
<td>Lyβ</td>
<td>3 to 1</td>
<td>102.572</td>
</tr>
<tr>
<td></td>
<td>Lyγ</td>
<td>4 to 1</td>
<td>97.254</td>
</tr>
<tr>
<td></td>
<td>Ly limit</td>
<td>∞ to 1</td>
<td>91.18</td>
</tr>
<tr>
<td>Balmer</td>
<td>Hα</td>
<td>3 to 2</td>
<td>656.281</td>
</tr>
<tr>
<td></td>
<td>Hβ</td>
<td>4 to 2</td>
<td>486.132</td>
</tr>
<tr>
<td></td>
<td>Hγ</td>
<td>5 to 2</td>
<td>434.048</td>
</tr>
<tr>
<td></td>
<td>H limit</td>
<td>∞ to 2</td>
<td>364.6</td>
</tr>
<tr>
<td>Paschen</td>
<td>Paα</td>
<td>4 to 3</td>
<td>1875.10</td>
</tr>
<tr>
<td></td>
<td>Paβ</td>
<td>5 to 3</td>
<td>1281.81</td>
</tr>
<tr>
<td></td>
<td>Paγ</td>
<td>6 to 3</td>
<td>1093.81</td>
</tr>
<tr>
<td></td>
<td>Pa limit</td>
<td>∞ to 3</td>
<td>820.4</td>
</tr>
</tbody>
</table>
Bohr model of Atom

Niels Bohr (1885-1962) proposed new atomic model in 1913.

• When an electron is in one of the quantized orbits, it does not emit any electromagnetic radiation. The electron can make a discontinuous transition from one stationary state to another. It emits or absorbs radiation during these jumps.

• Laws of classical mechanics apply to orbital motion of electrons in a stationary state, but not during a transition between states.

• When an electron makes a transition, the energy difference is released as a single photon of frequency, $\nu = E/h$.

• Permitted orbits are characterized by quantized values of the orbital angular momentum with $L = n h / 2\pi = n \hbar$ (\(\hbar\) is “h bar”).
Bohr model of Atom

Coulomb’s Law

\[ \vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r} \]

Proton has charge of \( q = +e = 1.602176462 \times 10^{-19} \text{ C} \)

Electron has charge of \( q = -e \)

This is a two-body problem, for which we can model as a reduced mass, \( \mu \), orbiting a total mass \( M \).

For this problem:

\[
\mu = \frac{m_e m_p}{m_e + m_p} = \frac{m_e (1836.15266 m_e)}{m_e + 1836.15266 m_e} = 0.999455679 m_e
\]

\[
M = m_e + m_p = (m_p/1836.15266) + m_p = 1.0005446 m_p
\]
Bohr model of Atom

Coulomb's Law

\[ \vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r} \]

Proton has charge of \( q = +e = 1.602176462 \times 10^{-19} \) C

Electron has charge of \( q = -e \)

\[ \vec{F} = \mu \vec{a} \rightarrow - \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r^2} \hat{r} = -\mu \frac{v^2}{r} \hat{r} \]

\[ \frac{1}{2} \times \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r^2} \times r = \frac{1}{2} \mu \frac{v^2}{r} \times r \rightarrow \frac{1}{8\pi\varepsilon_0} \frac{e^2}{r} = \frac{1}{2} \mu \frac{v^2}{r} = K \]

\[ U = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r} = -2K \]

Total Energy:

\[ E = K + U = K - 2K = -K = \frac{1}{2} U \]

Satisfies Virial Theorem! Electron-Proton system is a Bound Orbit.
Bohr model of Atom

\[ E = -K = -\frac{1}{2} \mu v^2 = -\frac{1}{8\pi\varepsilon_0} \frac{e^2}{r} \]

Bohr’s quantization of angular momentum gives

\[ L = \mu vr = n\hbar \]

Rewrite top equation in terms of angular momentum and then solve for \( r \):

\[ \frac{1}{8\pi\varepsilon_0} \frac{e^2}{r} = \frac{1}{2} \frac{\mu v^2}{\mu r^2} = \frac{1}{2} \frac{(\mu vr)^2}{\mu r^2} = \frac{1}{2} \frac{(n\hbar)^2}{\mu r^2} \]

\[ r = \frac{1}{2} \frac{(n\hbar)^2}{\mu} \times \frac{8\pi\varepsilon_0}{e^2} \]

\[ a_0 = \text{Bohr Radius} = 0.0529 \text{ nm} \]

Insert eqn. for \( r_n \) into equation for Energy gives:

\[ E = \frac{1}{2} U = \frac{1}{8\pi\varepsilon_0} \frac{e^2}{r} = \frac{1}{8\pi\varepsilon_0} \frac{e^2}{r} \times \frac{\mu e^2}{4\pi\varepsilon_0\hbar^2 n^2} \]

\[ E = \frac{\mu e^4}{32\pi^2\varepsilon_0^2\hbar^2 n^2} \frac{1}{n^2} = -13.6 \text{ eV} \frac{1}{n^2} \]

\( n \) is the principle quantum number
Emission Lines in Hydrogen

\[ E_{\text{photon}} = E_{\text{high}} - E_{\text{low}} \]

\[ E_{n=3} - E_{n=2} = -1.50 \text{ eV} - (-3.40 \text{ eV}) = 1.90 \text{ eV} \]

Wavelength of that photon is \( E = \frac{hc}{\lambda} \), or \( \lambda = \frac{hc}{E} = 656.3 \text{ nm} \).
Absorption Lines in Hydrogen

\[ E_{\text{photon}} = E_{\text{high}} - E_{\text{low}} \]

\[ E_{n=3} - E_{n=2} = -1.50 \text{ eV} - (-3.40 \text{ eV}) = 1.90 \text{ eV} \]

Wavelength of that photon is \( E = \frac{hc}{\lambda} \), or \( \lambda = \frac{hc}{E} = 656.3 \text{ nm} \).
Kirchhoff’s Laws:

• A hot, dense gas or hot solid object produces a continuous Spectrum with no dark spectral lines.

• A hot, diffuse (low density) gas produces bright spectral lines (emission lines).

• A cool, diffuse gas in front of a source of continuous spectrum produces dark spectral lines (absorption lines) in the continuous spectrum.
Kirchhoff’s Laws, Restated.

• A hot, dense gas or hot solid object produces a continuous spectrum with no dark spectral lines. This is blackbody radiation emitted at any temperature, $T > 0$ K, with a spectrum described by the Planck function $B_{\lambda}(T)$ and $B_{\nu}(T)$. The wavelength at which the Planck Function $B_{\lambda}(T)$ reaches its maximum is given by Wein’s displacement law.

• A hot, diffuse (low density) gas produces bright emission lines. Emission lines are produced when an electron makes a downward transition from a higher orbit to a lower orbit. The energy lost by then electron is carried away by a single photon.

• A cool, diffuse gas in front a source of continuous spectrum produces dark absorption lines in the continuous spectrum. Absorption lines are produced when an electron absorbs a photon with enough energy equal to the energy difference between the two transitions.
Spectrographs work on principle of Diffraction

\[ d \sin \theta = \begin{cases} 
  n\lambda & (n=0,1,2,3\ldots), \text{constructive interference} \\
  (n-1/2)\lambda & (n=0,1,2,3\ldots), \text{destructive interference}
\end{cases} \]

Double-Slit Experiment of Thomas Young (1773-1829)

\[
\lambda / \Delta \lambda = n \times N = \text{“Resolving Power”} = R
\]

\[
\Delta \lambda = \lambda / (n N)
\]

n = the “order” of the spectrum

Ability to resolve two wavelengths, separated by \( \Delta \lambda = |\lambda_1 - \lambda_2| \) is

Where \( N = \text{number of lines} \)
“Continuous” Spectra

Observed Spectra of
Vega-type Star
Solar-type Star

Here Stars Look almost exactly like blackbodies
Lots of absorption from atoms in the stars' atmospheres (more next week)

T = 10,000 K
T = 8000 K
T = 5800 K
T = 3000 K
Emission Line Galaxy

PN G000.2+06.1
Galaxy with Absorption and Emission Lines

![Graph showing absorption and emission lines in a galaxy. The y-axis represents intensity \( I_v \) in \( \mu Jy \), and the x-axis represents wavelength \( \lambda \) in Å. The graph includes various absorption and emission features labeled as low-ion IS abs, high-ion IS abs, stellar, nebular em, fine-structure em, and HI em/abs.]
Bohr’s model was successful, but did not account for all observed properties of atoms. This was a “semi-classical” picture of atoms as mini-solar systems. Full picture needed quantum mechanics.

de Broglie asked the question, “if photons sometimes act as particles, do particles sometimes act as waves?”

Answer led him to state:

\[ \nu = \frac{E}{h} \]
\[ \lambda = \frac{h}{p} = \frac{h}{(m \nu)} \]
Transition to Quantum Mechanical Picture of Atoms

Examples:
\[ \lambda = \frac{h}{p} = \frac{h}{(m \, v)} \]

1. electron moving at 1% the speed of light \((3 \times 10^6 \text{ m/s})\):

\[ \lambda = \frac{h}{(m_e \, v)} = \frac{(6.63 \times 10^{-34} \text{ J s})}{(9.11 \times 10^{-31} \text{ kg} \times 3 \times 10^6 \text{ m/s})} = 2.42 \times 10^{-10} \text{ m} = 0.242 \text{ nm} \]

2. jogging person with \(m = 80 \text{ kg}\) and \(v = 2 \text{ m/s}\):

\[ \lambda = \frac{h}{(m \, v)} = \frac{(6.63 \times 10^{-34} \text{ J s})}{(80 \text{ kg} \times 2 \text{ m/s})} = 4.14 \times 10^{-36} \text{ m} = 4.14 \times 10^{-27} \text{ nm} \]

Roughly 1 billionth of a billionth of the size of the nucleus of an atom!
Heisenberg’s Uncertainty principle.

One wave, well known wavelength, can determine momentum exactly by, $p = \frac{h}{\lambda}$.

But, because wave has infinite number of peaks, $x$ location is unknown.

Superposition of many waves with a combination of wavelengths.

$$\Psi = \sum_{n} \sin \left( \frac{2\pi x}{n\lambda} \right)$$

$\lambda$ not exactly known, $p$ has uncertainty. But, $x$ can be determined more accurately.

Transition to Quantum Mechanical Picture of Atoms
Transition to Quantum Mechanical Picture of Atoms
Heisenberg’s Uncertainty principle.

formally: $\Delta x \Delta p \geq \frac{\hbar}{2} \quad \Delta E \Delta t \geq \frac{\hbar}{2}$

in practice: $\Delta x \Delta p \approx \hbar \quad \Delta E \Delta t \approx \hbar$
Transition to Quantum Mechanical Picture of Atoms

Quantum Numbers

n: orbital state, determines energy

l: angular momentum state. Comes from true quantization of angular momentum, \( L = \sqrt{l(l+1)} \hbar \), where \( l = 0,1,2,\ldots,n-1 \). Historically, spectroscopic designations are s, p, d, f, g, h corresponding to \( l = 0,1,2,3,4,5 \).

\( m_l \): z-component of angular momentum. Seen when electric fields applied to atoms. Allowed values are all integers from -\( l \) to +\( l \).

\( m_s \): “Spin”. Intrinsic angular momentum of particles. Two types of particles. Fermions (electrons, protons, neutrons) with \( m_s = \pm \frac{1}{2} \). Bosons (photons) with \( m_s = \text{integers} \).
Quantum Numbers:  \( n, l, m_l, m_s \)

No two Fermions can have all the same quantum \# values. (This is important for atoms, but also for the structure of white dwarf and neutron stars.)

Selection Rules for atomic transitions:

Allowed transitions with \( \Delta l = \pm 1 \).

Allowed transitions called “Permitted”, occur on \( 10^{-8} \) s timescales.

Transitions with \( \Delta l \neq \pm 1 \) called “Forbidden” transitions. These do occur from metastable atomic states, but they need assistance (collisions between atoms, etc.)

Allowed transitions for Hydrogen
In atoms, electrons must orbit with integral number of wavelengths $n\lambda$ where $\lambda$ is the de Broglie wavelength. Else, the electron wave will be out of phase with itself and destructively interfere.
Radioactivity

• In 1896 Antoine Henri Becquerel discovered that Uranium would fog a photographic plate, and that the radiation emitted by Uranium could be deflected in a magnetic field. (Unlike Marie Curie’s discovery of X-ray radiation.)

• Rutherford subsequently identified two kinds of radiation from Uranium, alpha, and beta radiation. Paul Villard discovered that another element, radium, gave off a third type of radiation particle with no charge, named gamma.

• Today, we understand these radiation originate from an unstable nucleus that emits a Helium nucleus (alpha particle), an electron (beta particle) or a positron (positive beta particle), or a high-energy photon (gamma particle).

• Becquerel shared the Nobel Prize with Marie Curie in 1903 for their discoveries of emission.

Edward Rutherford
(1871-1937)

Antoine Henri Becquerel
(1852-1908)
Radioactivity

Notation for the number of nucleons of a particular atom:

\[ \frac{A}{Z}X \]

- \( X \) is the chemical symbol (H, He, C, O, N, Fe, ...)
- \( Z \) is the number of protons (and total charge)
- \( A \) is the number of nucleons (neutrons + protons)

Helium with 2 protons and 2 neutrons is: \( ^4_2\text{He} \)

**Alpha Decay**:

\[
\frac{A}{Z}X \rightarrow \frac{A-4}{Z-2}Y + \frac{4}{2}\text{He}
\]

**Beta Decay**:

\[
\frac{A}{Z}X \rightarrow \frac{A}{Z+1}Y + e^- + \bar{\nu}_e
\]

\[
\frac{A}{Z}X \rightarrow \frac{A}{Z-1}Y + e^+ + \nu_e
\]

**Gamma Radiation**:

\[
\frac{A}{Z}X \rightarrow \frac{A}{Z}X + \gamma
\]
Radioactivity

Radioactive Decay is a Statistical process, so it must be proportional to the number of atoms in a sample:

$$\frac{dN}{dt} = -\lambda N$$

One can show that if $N(t) = N_0 e^{-\lambda t}$ then

$$\frac{d}{dt} N(t) = N_0 \frac{d}{dt} e^{-\lambda t} = -N_0 \lambda e^{-\lambda t} = -\lambda N$$

$N_0$ is the original number in a sample of atoms. $\lambda$ is related to the “half-life”, $\tau_{1/2}$, by

$$\lambda = \frac{\ln 2}{\tau_{1/2}}$$

The half-life is the time it takes for the number of atoms you have to be reduced by a factor of 2. Alternatively, there is a 50% chance that any individual atom will decay in a time $\tau_{1/2}$. 
Radioactivity

**Radioactive Dating**: You can use radioactive isotopes and compare them to stable isotopes to derive the age of that sample of atoms.

This has been done with Earth Rocks, Moon Rocks, Asteroids. (Also Carbon-dating, used to date age of fossils, for example.)

Suppose an isotope type A decays into isotope B (which is stable) then after some time t

\[ N_{A,f} = N_{A,i} e^{-\lambda t} \]

where \( N_{A,i} \) is the initial number and \( N_{A,f} \) is the final number.

The total number of atoms of A and B must remain constant, so:

\[ N_{A,f} + N_{B,f} = N_{A,i} + N_{B,i} \]

Solving for \( N_{A,i} \) yields a formula for the change in the number of atoms of B:

\[ N_{B,f} - N_{B,i} = (e^{\lambda t} - 1)N_{A,f} \]

In practice you study the ratio of B and A to a third (stable and constant) element C:

\[ \frac{N_{B,f}}{N_C} = (e^{\lambda t} - 1)\frac{N_{A,f}}{N_C} + \frac{N_{B,i}}{N_C} \]
Radioactivity

\[ \frac{N_{B,f}}{N_C} = (e^{\lambda t} - 1) \frac{N_{A,f}}{N_C} + \frac{N_{B,i}}{N_C} \]

One plots the relative abundances of the stable product versus the relative abundances of the radioactive isotope in the sequence at various locations in the rock. The slope \( m = (e^{\lambda t} - 1) \) is directly related to the age of the sample.

Data from the Moon looks at the decay of rubidium-87 to strontium-87. Using the plot \( m = (e^{\lambda t} - 1) = 0.0662 \).

We know that \( \lambda = 0.0146 \times 10^{-9} \text{ yr}^{-1} \) for rubidium-87, so we can solve for \( t = 4.39 \times 10^9 \text{ yr} \).
Radioactivity

Radioactive Decay is a Statistical process, so it must be proportional to the number of atoms in a sample

“Carbon dating” measures the ratio of Carbon-14 in a substance to that in the Earth’s atmosphere. Carbon-14 is unstable and has a half-life of 5730 years. It decays by the following:

\[ ^{14}_6 \text{C} \rightarrow ^{14}_7 \text{N} + e^- + \bar{\nu}_e \]
Radioactivity

An example is the natural decay chain of uranium-238 which is as follows:

- decays via \( \alpha \)-emission, with a half-life of 4.5 billion years to thorium-234
- decays via \( \beta \)-emission, with a half-life of 24 days to protactinium-234
- decays via \( \beta \)-emission, with a half-life of 1.2 minutes to uranium-234
- decays via \( \alpha \)-emission, with a half-life of 240 thousand years to thorium-230
- decays via \( \alpha \)-emission, with a half-life of 77 thousand years to radium-226
- decays via \( \alpha \)-emission, with a half-life of 1.6 thousand years to radon-222
- decays via \( \alpha \)-emission, with a half-life of 3.8 days to polonium-218
- decays via \( \alpha \)-emission, with a half-life of 3.1 minutes to lead-214
- decays via \( \beta \)-emission, with a half-life of 27 minutes to bismuth-214
- decays via \( \beta \)-emission, with a half-life of 20 minutes to polonium-214
- decays via \( \alpha \)-emission, with a half-life of 160 microseconds to lead-210
- decays via \( \beta \)-emission, with a half-life of 22 years to bismuth-210
- decays via \( \beta \)-emission, with a half-life of 5 days to polonium-210
- decays via \( \alpha \)-emission, with a half-life of 140 days to lead-206, which is a stable nuclide.