White Dwarf Stars

Sirius A
brightest star in the sky
m = -1.46.

In 1844, Friedrich Bessel deduced it was a binary.
In 1862 Alvan Graham Clark discovered the companion.

Sirius B
m = 8.30
Stellar Deaths and Compact Objects

**White Dwarf Stars:** Surface temperatures range from 5,000 to 80,000 K. They are the exposed stellar “cores” containing the products of nuclear fusion. They are composed primarily of Carbon-Oxygen and Helium. They are made by stars with initial mass $<8-9$ solar masses.

Their masses range from $\sim0.4$ to 1.4 solar masses. Mass distribution of “DA” white dwarfs is peaked at 0.56 solar masses (80% have masses between 0.42 and 0.70 solar masses).

Different classes:

**DA white dwarfs:** (2/3 of all WDs, including Sirius B) have hydrogen absorption lines.

**DB white dwarfs:** (8% of all WDs) No H lines, only He absorption lines.

**DC white dwarfs:** (14%) no lines at all, only continuum in their spectrum.

**DD white dwarfs:** Carbon features seen.

**DZ:** Other metal lines seen.
Stellar Deaths and Compact Objects

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Stellar Deaths and Compact Objects

\[ \log_{10}(L/L_\odot) \]

against

\[ T_e (\text{K}) \]

with points labeled B7, B8, B9, and A0.
Stellar Deaths and Compact Objects

There is no nuclear fusion in White Dwarfs. What prevents gravity from collapsing all the matter to a point source?

The answer is Degeneracy Pressure. The electrons in the core are fermions. Thus, at most one electron can be in each unique quantum state.

As the temperature of the core cools, the “fermions” will fill up available quantum states with the lowest energies.

As the temperature cools further, the fermions in excited states can not drop to lower states and this produces a “pressure” in the fermionic gas.

At T=0 K all the lower energy states and none of the higher states are occupied.

Such a gas is completely degenerate!
Stellar Deaths and Compact Objects

Estimate Degeneracy Pressure Using:

1. Exclusion principle for Fermions: at most 1 electron can be in each quantum state.

2. Heisenberg Uncertainty Principle: \( \Delta x \Delta p_x \approx \hbar/2\pi \). The more closely confined (packed) electrons will have greater momentum.

\[
P \approx \frac{1}{3} \int_0^\infty n_p p v \, dp \approx \frac{1}{3} n_e p v
\]

In a degenerate gas, the electrons are packed as tightly as possible. Separation is \( \Delta x \approx n_e^{-1/3} \). And \( p_x \approx \Delta p_x \approx \hbar/(2\pi \Delta x) \)

All dimensions are equally likely, \( p^2 = p_x^2 + p_y^2 + p_z^2 = 3p_x^2 \)

\[p = (3)^{1/2} p_x = (3)^{1/2} \frac{\hbar n_e^{1/3}}{2\pi}\]
Stellar Deaths and Compact Objects

For a core that is fully ionized, the number density of free electrons is

\[ n_e = \left( \frac{\text{# electrons}}{\text{nucleon}} \right) \left( \frac{\text{# nucleons}}{\text{Volume}} \right) = \left( \frac{Z}{A} \right) \frac{\rho}{m_H} \]

Here \( m_H \approx m_p \approx m_n \)

\[ p \approx \sqrt{3\hbar} \left[ \left( \frac{Z}{A} \right) \frac{\rho}{m_H} \right]^{1/3} \]

Rewrite using the velocity, \( v = \frac{p}{m_e} \approx (3)^{1/2} \frac{\hbar}{(2\pi m_e)} n^{1/3} \)

\[ v \approx \frac{\sqrt{3\hbar}}{m_e} \left[ \left( \frac{Z}{A} \right) \frac{\rho}{m_H} \right]^{1/3} \]

\[ P \approx \frac{\hbar^2}{m_e} \left[ \left( \frac{Z}{A} \right) \frac{\rho}{m_H} \right]^{5/3} \]
Stellar Deaths and Compact Objects

More exactly:

\[ P = \frac{(3\pi)^{2/3}}{5} \frac{\hbar}{m_e} \left( \frac{Z}{A} \right) \frac{\rho}{m_H} \left( \frac{Z}{A} \right) \left( \frac{\rho}{m_H} \right)^{5/3} \]

For \( Z/A = 0.5 \) (a Carbon-Oxygen White Dwarf): \( P_C = 1.9 \times 10^{22} \) N m\(^{-2}\)

Compare to pressure from Gravity for hydrostatic equilibrium, for the (unrealistic) assumption of constant density:

\[ \frac{dP}{dr} = -\frac{GM_r\rho}{r^2} = -\frac{G(\frac{4}{3}\pi r^2 \rho)\rho}{r^2} = -\frac{4}{3} \pi G \rho^2 r. \]

Integrating with the condition that \( P=0 \) at the surface:

\[ P(r) = \frac{2}{3} \pi G \rho^2 (R^2 - r^2) \]

At \( r=0 \) we have the central pressure:

\[ P_C \approx \frac{2}{3} \pi G \rho^2 R_{WD}^2 \]

\( P_C = 3.8 \times 10^{22} \) N m\(^{-2}\) for Sirius B (\( R=0.008 \) R\(_\odot\), \( M=1.0 \) M\(_\odot\)).

The Electron Degeneracy Pressure is nearly that of Gravity (they balance)!
Stellar Deaths and Compact Objects

In 1931, at the age of 21 the Indian physicist Subrahmanyan Chandrasekhar worked out that White Dwarf stars have a maximum mass: The Chandrasekhar Limit.

The Mass-Volume Relation. Set Central pressure equal to the degeneracy pressure:

\[
\frac{2}{3} \pi G \rho^2 R_{WD}^2 = \frac{(3\pi)^{2/3}}{5} \frac{\hbar}{m_e} \left[ \left( \frac{Z}{A} \right) \frac{\rho}{m_H} \right]^{5/3}
\]

Solve for \( R_{WD} \) assuming constant density:

\[
R_{WD} = \frac{(18\pi)^{2/3}}{10} \frac{\hbar^2}{G m_e M_{WD}^{1/3}} \left[ \left( \frac{Z}{A} \right) \frac{1}{m_H} \right]^{5/3}
\]

Depends only on the mass! For 1 \( M_\odot \) Carbon-Oxygen White Dwarf, \( R \approx 2.9 \times 10^6 \) m, 50% the size of the Earth!

Note that \( R \times M^{1/3} = \) constant, or \( V \times M = \) constant.

As \( M \) goes up, \( R \) goes down!
Stellar Deaths and Compact Objects

The Chandrasekhar Limit.

In extreme limit, let $v=c$ (velocity of electrons increases to maintain degeneracy pressure). Setting degeneracy pressure for this case equal to the gravitational pressure:

$$P = \frac{3\pi^2}{4} \frac{\hbar c}{\rho} \left( \frac{Z}{A} \right)^{4/3} = \frac{2}{3} \pi G \rho^2 R_{WD}^2$$

Solving for the Mass:

$$M_{\text{Ch}} = \frac{3\sqrt{2}}{8} \left( \frac{\hbar c}{G} \right)^{3/2} \left[ \left( \frac{Z}{A} \right) \frac{1}{m_H} \right]^2 = 0.44 M_\odot$$

Solving using a full relativistic treatment gives the answer Chandrasekhar got:

$M_{\text{Ch}} = 1.44 M_\odot$. The Chandrasekhar Limit. No White Dwarf has yet been found with a mass greater than this!
Stellar Deaths and Compact Objects

The Chandrasekhar Limit.

![Graph showing the relationship between mass and radius for compact objects, with a point labeled Sirius B.]
Stellar Deaths and Compact Objects

The cooling of White Dwarfs. No energy source, WDs cool over time:

Sudden drop implies that White Dwarfs have only been forming over the last 9.3 billion years ago.
Stellar Deaths and Compact Objects

First recorded “New Star” is likely Supernova 1006 (SN 1006), reported by astronomers in China, England, Japan, Egypt, and Iraq. Appeared on April 30, 1006 and faded from around one year later.

Next one occurred on July 4, 1054, recorded by Chinese, Japanese, and Korean scholars. This supernovae remnant is now the Crab nebula, roughly 2000 pc away in the constellation Taurus.
Stellar Deaths and Compact Objects

Crab Supernova explosion
Stellar Deaths and Compact Objects

Next reported Supernovae witnessed by Tycho Brahe in 1572 and Johannes Kepler in 1604. This was the last known supernovae in our Galaxy.

Kepler’s SN Remnant
Stellar Deaths and Compact Objects

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In 1987, the world witnessed SN 1987A in the Large Magellanic Cloud, which occurred in a massively star-forming region. This was the first “modern-era” supernovae that was studied extensively.

SN 1987A made Time Magazine!
Mass loss in late evolutionary stages

\[ 0.1 \text{ pc} = 20,000 \text{ AU} \]
\[ = 1000 \times \text{the semi-major axis of Neptune} \]

\( \eta \) Carinae, a LBV, has a mass of \( \sim 120 \, M_\odot \), rapidly losing mass to stellar winds.
NGC 1058
D = 8.4 Mpc (24 x 10^6 lyr)

NGC 1058 with SN 2007gr

on 2003 December 24
with the 4.2m William Herschel Telescope (+PFIP)

on 2007 August 26
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Stellar Deaths and Compact Objects

**Type I**: Supernovae with no hydrogen lines in their spectra

- **Type Ia** have a strong Si II line at 615 nm. **Type Ib** have strong helium lines, **Type Ic** have no helium.

**Type II**: Supernovae with hydrogen lines in their spectra

Supernova Classification Scheme
(spectra at maximum light)

- **Type I**: no H lines
  - **Si II lines**: Type Ia
  - no Si II lines: Type Ib

- **Type II**: H lines
  - Plateau
    - **He lines**: Type Ib
    - no He lines: Type Ic
  - no Plateau
    - **Type II-P**
    - **Type II-L**
Stellar Deaths and Compact Objects

Core-Collapse Supernovae

Post-main sequence stars more massive than 8 solar masses have a very different fate than lower mass stars. Details are still not fully understood.
Stellar Deaths and Compact Objects

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\[
\begin{align*}
\frac{28}{14}\text{Si} + \frac{4}{2}\text{He} & \iff \frac{32}{16}\text{S} + \gamma \\
\frac{32}{16}\text{S} + \frac{4}{2}\text{He} & \iff \frac{36}{18}\text{Ar} + \gamma \\
& \quad \vdots \\
\frac{52}{24}\text{Cr} + \frac{4}{2}\text{He} & \iff \frac{56}{28}\text{Ni} + \gamma \\
\frac{52}{24}\text{Cr} + \frac{4}{2}\text{He} & \iff \frac{56}{26}\text{Fe} + \gamma
\end{align*}
\]

Any reactions beyond Fe-56 are endothermic: they remove energy from the star. Because heavy-element fusion moves closer to “Iron peak”, the binding energy released is less and less, so the timescales for each reaction sequence is shorter and shorter.

For a 20 M$_\odot$ star, H-burning takes $10^7$ yr, He-burning takes $10^6$ yr, C-burning takes 300 years, O-burning takes 200 days, and Si-burning takes 2 days!
Stellar Nucleosynthesis

Binding Energy per Nucleon

Binding energy is the amount of energy that was released in the creation of an element. Recall that the fusing of 4 Hydrogen atoms into Helium releases $\Delta m = 0.028697 \text{ u}$, or an energy of $E = \Delta mc^2 = 26.731 \text{ MeV}$.

$E_b = \Delta mc^2 = [ Z m_p + (A - Z) m_n - m_{\text{nucleus}} ] c^2$
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\end{align*}
\]

At very high temperatures, photons have enough energy to break up nuclei (photodisintegration). For example:

\[
\begin{align*}
\frac{52}{26}\text{Fe} + \gamma & \rightarrow 13\frac{4}{2}\text{He} + 4n \\
\frac{4}{2}\text{He} + \gamma & \rightarrow 2\, p^+ + 2\, n
\end{align*}
\]

This also removes energy from the star and undoes all the nucleosynthesis!
Stellar Deaths and Compact Objects

Core-Collapse Supernovae

Under extreme circumstances, like those at the cores of massive stars, the free electrons are captured by heavy nuclei and convert protons to neutrons:

\[ p^+ + e^- \rightarrow n + \nu_e \]

For a 15 M\(_\odot\) star, this occurs when \( T_C \sim 8 \times 10^9 \) K and \( \rho_C \sim 10^{13} \) kg/m\(^3\). The amount of energy carried away by neutrinos is enormous, roughly 10,000,000 x that of the photon luminosity! (This might cause the explosion.)

The remnant is a neutron star for stars with 8 M\(_\odot\) < M < 25 M\(_\odot\) supported by the degeneracy pressure of the neutrons. Higher-yet-mass stars probably leave behind black holes.
Stellar Deaths and Compact Objects

Core-Collapse Supernovae

Supernovae deliver much of the carbon, nitrogen, oxygen, neon, etc. back to the galaxy.

**s-Process and r-Process element formation.**

In addition, elements with $Z > 26$ do not form from He-nuclei capture due to the increasing Coulomb barrier. However, free neutrons do make it into the nuclei of heavy elements, causing nucleosynthesis. The reaction is:

$$\frac{A}{Z}X + n \rightarrow \frac{A+1}{Z}X + \gamma$$

Where the new element may decay via the beta-decay:

$$\frac{A}{Z}X + n \rightarrow \frac{A+1}{Z+1}X + e^- + \bar{\nu}_e + \gamma$$

Elements with short beta-decay half-lives form via **slow (s-)** process reactions. These tend to form more stable nuclei. If the half-live is long, the process is a **rapid (r-)** reaction and these produce neutron-rich nuclei. s-processes occur during stellar nucleosynthesis, whereas r-process reactions occur when the neutrino flux is higher... during the Supernova explosion.
Stellar Deaths and Compact Objects

Core-Collapse Supernovae

SN 1987A was a “Type II P” event. These are most common core-collapse supernovae.

The energy source of the fading “afterglow” following the supernovae (called the “lightcurve”) is the radioactive decay of $^{56}\text{Ni}$ to $^{56}\text{Co}$ through beta-decay with a half-life $\tau_{1/2}=6.1$ days:

$$^{56}\text{Ni} \rightarrow ^{56}\text{Co} + e^+ + \nu_e + \gamma$$

$^{56}\text{Co}$ is also radioactive and decays to $^{56}\text{Fe}$ with $\tau_{1/2}=77.7$ days.

$$^{56}\text{Co} \rightarrow ^{56}\text{Fe} + e^+ + \nu_e + \gamma$$
The decay of $^{56}\text{Co}$ powers most of the light curve (afterglow of explosion).

The afterglow of SN1987A is explained by the decay of about 0.075 $M_\odot$ of $^{56}\text{Co}$ (and other elements) produced during the explosion.
Stellar Deaths and Compact Objects

Neutron Stars: Degeneracy

\[ R_{NS} \approx \frac{(18\pi)^{2/3}}{10} \frac{\hbar^2}{GM_{NS}^{1/3}} \left( \frac{1}{m_H} \right)^{8/3} \]

Neutron stars have a degeneracy pressure just like White Dwarfs. Derivation is similar. A 1.4 solar mass Neutron Star is basically a single nucleus containing \( 1.4 \, M_\odot \) / \( m_n \sim 10^{57} \) neutrons.

It could be considered a single atom with \( Z=0 \) and \( A=10^{57} \)!

Like White Dwarfs, we have that \( R \times M^{1/3} = \text{constant} \). Or, \( M \times V = \text{constant} \).

Including effects of special and general relativity, and rotation, etc, the maximum possible mass for a neutron star is 2.2 \( M_\odot \) (non-rotating) or 2.9 \( M_\odot \) (rotating rapidly). Beyond this mass is nothing but a black hole.

( There might be a thing as a quark star composed of neutrons and strange quarks, which could reach a higher mass... but we’re getting into science fiction. )
Stellar Deaths and Compact Objects

Supernovae Type Ia are not core-collapse

These are special, and probably result from when a white dwarf forms in a binary star system. The WD counterpart eventually becomes a giant star and sheds its outer layers onto the WD. The WD eventually grows larger than $1.4 \, M_\odot$ and becomes a neutron star. The explosion is a Type Ia supernova.
Because we think all Supernovae Type Ia have the same origin, their explosions should be similar. Empirically, supernovae type Ia have remarkably consistent energy outputs. Only difference is that brighter SN Ia have longer decay time:

They are now being used as “standard candles”. If you can measure the decay time of the Supernovae explosion (how long it takes to fade), then you know its luminosity.
Pulsars = Rapidly Rotating Neutron Stars?

Maximum angular velocity would be found by equating the centripetal and gravitational accelerations. $\omega^2 R = G M / R^2$. Using $P = 2\pi / \omega$, we get $P = 2\pi (R^3 / GM)^{1/2}$.

For white dwarfs, $P \sim 7$ s. For a 1.4 solar mass neutron star, $P \sim 5 \times 10^{-4}$ s.

This is the rough range of possible rotation periods.

In 1967, Jocelyn Bell tuned a radio telescope to 81.5 MHz. She discovered a radio source with a period of $P=1.337$ s. Because it was so precise, one of the first interpretations was that this was a signal beacon from aliens....

More pulsars were quickly found. There are currently >1500 known.

Most pulsars are 0.25 to 2 s in period. Longest is 11.8s and fastest is 0.00139 s.

Extremely well defined pulse periods and make accurate clocks. Most precise is known to be 0.00155780644887275 s for PSR 1937+214.

Periods of all pulsars increase as pulses gradually slow down. Typical rates are $dP/dt \sim 10^{-15}$. Lifetimes are a few $\times 10^7$ yrs.

Some pulsars have “gitches”, which change their period suddenly by one part in $\sim 10^{-6}$ every few years. These are sudden spin-ups.
Pulsars = Rapidly Rotating Neutron Stars?
In the 1960s, Russell Hulse and Joseph Taylor discovered that the pulsar PSR 1913+16 was a binary system of two neutron stars (or a neutron star and a white dwarf). The orbital separation of the system is a little larger than the Sun’s diameter.

A 30-year study of this system showed that the orbital period is speeding up as a result of gravitational radiation (gravity waves!). The two stars should merge in ~300 Myr.

Theoretical : $\frac{dP}{dt} = -(2.40242 \pm 0.0000002) \times 10^{-12}$.

Measured: $\frac{dP}{dt} = -(2.4056 \pm 0.0051) \times 10^{-12}$. Excellent!

Hulse and Taylor received the Nobel prize in 1993 for this work.