Lecture 5: Newton’s Laws

Astronomy 111
Isaac Newton (1643-1727): English

Discovered:
three laws of motion,
one law of universal gravitation.
Newton’s great book:

Newton’s laws are *universal* in scope, and *mathematical* in form.
Newton was a great scientist

• Newton realized that gravity worked in the same way in the heavens as on earth!

• Therefore, the same physical laws operating on earth also operate in the heavens ➜ one Universe

• Newton revolutionized math (invented calculus) and science (experiments with gravity, light, etc).
Newton’s First Law of Motion:

An object remains at rest, or moves in a straight line at constant speed, unless acted on by an outside force.

Precise mathematical laws require **precise** definitions of terms:

SPEED = rate at which an object changes its position.
Example: 65 miles/hour.

VELOCITY = speed plus direction of travel.
Example: 65 miles/hour to the north.
Acceleration = rate at which an object changes its velocity.

Acceleration can involve:
(1) increase in speed
(2) decrease in speed
OR
(3) change in direction
Force

Force = a push or pull acting on an object

Examples:
- gravity = pull
- electrostatic attraction = pull
- electrostatic repulsion = push
Newton’s Second Law of Motion

The acceleration of an object is directly proportional to the force acting on it, and inversely proportional to its mass.

In mathematical form: \( a = \frac{F}{m} \)  or  \( F = ma \)

Or alternatively: \( F = G\left(\frac{Mm}{r^2}\right) \)
Example: Newton’s Second Law

A package of cookies has mass

\[ m = 0.454 \text{ kilograms}, \]

And experiences gravitational acceleration

\[ g = 9.8 \text{ meters/second}^2 \]

How large is the force acting on the cookies?

\[ F = m \times a \]

\[ F = (0.454 \text{ kg})(9.8 \text{ m/sec}^2) \]

\[ F = 4.4 \text{ kg m/sec}^2 = 4.4 \text{ Newtons} \]

(4.4 Newtons = 1 pound)
Newton’s Third Law of Motion

For every action, there is an equal and opposite reaction.

Whenever A exerts a force on B, B exerts a force on A that’s equal in size and opposite in direction.

All forces come in pairs.
Example: Newton’s Third Law

Cookies push on hand: \( F = 1 \) pound, downward.
Hand pushes on cookies: \( F = 1 \) pound, upward.

Remove hand!

Earth pulls on cookies: \( F = 1 \) pound, downward.
Cookies pull on earth: \( F = 1 \) pound, upward.
THIRD Law states:
force on Earth = force on cookies
SECOND Law states:
acceleration = force divided by mass
Mass of Earth = $10^{25} \times$ mass of cookies
Therefore, acceleration of cookies = $10^{25} \times$ acceleration of Earth.
(Cookies reach a high speed while the Earth hardly budges.)
Second and Third Laws

But...why do the cookies and the Earth exert a force on each other?

Newton’s Law of Gravity states that gravity is an attractive force acting between ALL pairs of massive objects.

Gravity depends on:
(1) MASSES of the two objects,
(2) DISTANCES between the objects.
Newton’s Law of Gravity

The gravitational force between two objects

\[ F = G \left( \frac{Mm}{r^2} \right) \]

- \( F \) = gravitational force
- \( M \) = mass of one object
- \( m \) = mass of the second object
- \( r \) = distance between centers of objects
- \( G \) = “universal constant of gravitation”
Newton’s Law of Gravity

- Every mass attracts every other mass through gravity

- Strength of the gravitational attraction is directly proportional to the product of their masses (more mass, more gravitational attraction)

- Strength of the gravitational attraction decreases with the square of the distance between the centers of the objects (inverse square law)
Gravitational force varies **directly** with mass and **inversely** with square of distance.

Double the distance between objects: Force 1/4 as large.  
Triple the distance between objects: Force 1/9 as large.

\[ G = 6.7 \times 10^{-11} \text{ Newtons m}^2 / \text{kg}^2 \text{ (very small!)} \]
Example: What is gravitational force between Earth and cookies?

\[ F = G \left( \frac{Mm}{r^2} \right) \]

- \( M \) = mass of Earth = \( 6.0 \times 10^{24} \) kg
- \( m \) = mass of cookies = 0.454 kg
- \( r \) = radius of Earth = \( 6.4 \times 10^{6} \) m
- \( G \) = \( 6.7 \times 10^{-11} \) Newtons m\(^2\) / kg\(^2\)

Let's plug the numbers in:

\[ F = 4.4 \text{ Newtons} = 1 \text{ pound} \]
Example Encore: What is acceleration of cookies?

Second Law of Motion:

\[ a = \frac{F}{m} \]

Law of Gravity:

\[ F = G\left(\frac{Mm}{r^2}\right) \]

Therefore...

\[ a = G\left(\frac{Mm}{r^2}\right) \frac{1}{m} = \frac{GM}{r^2} = 9.8 \text{ m/sec}^2 \]

Independent of mass of cookies!!
Newton’s question: can GRAVITY be the force keeping the Moon in its orbit?

Newton’s approximation: Moon is on a circular orbit.

Even if its orbit were perfectly circular, the Moon would still be accelerated.
Example: the Moon’s orbital speed

radius of orbit: \( r = 3.8 \times 10^8 \text{ m} \)

circumference of orbit: \( 2\pi r = 2.4 \times 10^9 \text{ m} \)

orbital period: \( P = 27.3 \text{ days} = 2.4 \times 10^6 \text{ sec} \)

orbital speed:

\[
v = \frac{2\pi r}{P} = 10^3 \text{ m/sec} = 1 \text{ km/sec!}
\]
The required acceleration is:

$$a = \frac{v^2}{r}$$

$v = \text{orbital speed}$, $r = \text{orbital radius}$

For the Moon:

$$v = 10^3 \text{ m/sec}$$

$$r = 3.8 \times 10^8 \text{ m}$$

$$a = \frac{v^2}{r} = \frac{(10^3 \text{ m})^2}{3.8 \times 10^8 \text{ m}} = 0.0026 \text{ m/sec}^2$$
Acceleration provided by gravity

\[ a = \frac{GM}{r^2} \]

At the surface of the Earth \((r = \text{radius of Earth})\)

\[ a = 9.8 \text{ m/sec}^2 \]

At the orbit of the Moon \((r = 60 \times \text{radius of Earth})\)

\[ a = \frac{9.8 \text{ m/sec}^2}{(60)^2} = \frac{9.8 \text{ m/sec}^2}{3600} \]

\[ a = 0.0027 \text{ m/sec}^2 \]

(close enough, given rounding errors)
Bottom Line

If gravity goes as one over the square of the distance,
then it provides the right acceleration to keep the Moon on its orbit ("to keep it falling").

Triumph for Newton!!
Newton modified, expanded Kepler’s Laws of Motion

Kepler’s First Law:
The orbits of the planets around the Sun are ellipses with the Sun at one focus.

Newton’s revision:
The orbits of any pair of objects are conic sections with the center of mass at one focus.
As the Earth pulls on Moon, Moon pulls on Earth

Both Earth and Moon orbit the center of mass of the Earth-Moon system:

Center of mass = balance point: closer to more massive object.
Artificial satellites as envisaged by Isaac Newton:

To put an object into orbit, launch it sideways with a large enough speed.

How large is large enough?
The shape of the orbit depends on the speed of the satellite at launch:

Low speed = *closed* orbit, a circle or ellipse.
High speed = *open* orbit, a parabola or hyperbola.

Circles, ellipses, parabolas, and hyperbolas are called *conic sections.*
To remain in a circular orbit just above the Earth’s surface, a satellite must have $v = 7.9$ km/sec.

To attain an open orbit, a satellite must reach at least $11.2$ km/sec.
Some extra math:

First cosmic velocity:

\[ \frac{mv^2}{r} = G \frac{Mm}{r^2} \Rightarrow v_{1st} = \sqrt{\frac{GM_E}{R_E}} \]

\[ v_{1st} = 7.9 \text{ km/sec} \]

Second (escape) cosmic velocity:

\[ \frac{mv^2}{2} = G \frac{Mm}{r} \Rightarrow v_{2nd} = \sqrt{\frac{2GM_E}{R_E}} \]

\[ v_{2nd} = 11.2 \text{ km/sec} = \sqrt{2} \times v_{1st} \]
Kepler’s Second Law

A line from a planet to the Sun sweeps out equal areas in equal time intervals.

Newton’s revision:
Angular momentum conserved.
The product of the orbital speed \( (v) \) and the distance from the center of mass \( (r) \) is constant:

\[ v \times r = \text{const} \]

As \( r \) increases, \( v \) must decrease.
angular momentum = \( m \times v \times r \)
Kepler’s Third Law:

\[ P^2 = a^3 \]

\( P = \) orbital period (in years)
\( a = \) semimajor axis (in A.U.)

Newton's revision:

\[ P^2 = \left[ \frac{4\pi^2}{G(M + m)} \right] a^3 \]

\( P = \) orbital period (in seconds)
\( a = \) semimajor axis (in meters)
\( G = \) universal constant of gravitation
\( M = \) mass of one object (in kilograms)
\( m = \) mass of other object (in kilograms)
Newton's revision at work:

\[
P = \sqrt{\frac{4\pi^2}{G(M + m)}} a^3
\]

For the Earth's orbit:

\[M = 2 \times 10^{30} \text{ kg (mass of the Sun)}\]
\[a = 1.5 \times 10^{11} \text{ m (1 A.U.)}\]
\[G = 6.67 \times 10^{-11} \text{ m}^3 / \text{kg sec}^2\]

\[\Rightarrow P = 3.15 \times 10^7 \text{ sec!}\]
Kepler’s third law applies only to objects orbiting the Sun. Newton’s revision applies to all pairs of object orbiting each other.

\[ M + m = \frac{4\pi^2 a^3}{GP^2} \]

Newton’s revision can be used to find masses of distant objects (e.g., binary stars).
Kepler described **how** planets move; Newton explained **why** they move that way.

Kepler’s laws result naturally from Newton’s laws of motion and Newton’s law of gravity. Kepler’s laws of planetary motion, as modified by Newton, are **universal**.
In-class assignment

Two identical robotic spacecraft are launched by NASA. The first is inserted into a circular orbit 20,000 km from the center of the Earth, while the second flies to the planet Mongo and is inserted into a circular orbit that is 20,000 km from the center of Mongo. However, the spacecraft orbiting Mongo has an orbital period exactly \( \frac{1}{4} \) the period of the spacecraft orbiting Earth.

What is the mass of Mongo in units of the mass of the Earth?

(Hint: you do not need to know G or the mass of the Earth to do this problem and you can assume the masses of both spacecraft are negligibly small compared to the masses of both Earth and Mongo)