Newton’s Laws

One day, Newton was not fast enough, but he managed to confuse the villager with some nonsense about a mystical force pulling apples toward Hell.
Isaac Newton (1643-1727): English

Discovered:
three laws of motion,
one law of universal gravitation.
Newton’s great book:

Newton’s laws are *universal* in scope, and *mathematical* in form.
Newton’s First Law of Motion:

An object remains at rest, or moves in a straight line at constant speed, unless acted on by an outside force.

Precise mathematical laws require **precise** definitions of terms:

**SPEED** = rate at which an object changes its position.

Example: 65 miles/hour.

**VELOCITY** = speed plus direction of travel.

Example: 65 miles/hour to the north.
Acceleration

= rate at which an object changes its velocity.

Acceleration can involve:
(1) increase in speed
(2) decrease in speed
OR
(3) change in direction.
Force

= a push or pull acting on an object.

Examples:
gravity = pull
electrostatic attraction = pull
electrostatic repulsion = push
Newton’s Second Law of Motion:

The acceleration of an object is directly proportional to the force acting on it, and inversely proportional to its mass.

In mathematical form: \[ a = \frac{F}{m} \]

Or alternatively: \[ F = G \left( \frac{Mm}{r^2} \right) \]
Example of Newton’s Second Law:

A package of cookies has mass
\[ m = 0.454 \text{ kilograms}, \]
And experiences gravitational acceleration
\[ g = 9.8 \text{ meters/second}^2 \]
How large is the force acting on the cookies?

\[ F = m \times a \]

\[ F = (0.454 \text{ kg})(9.8 \text{ m/sec}^2) \]

\[ F = 4.4 \text{ kg m/sec}^2 = 4.4 \text{ Newtons} \]

(4.4 Newtons = 1 pound)
Newton’s Third Law of Motion:

For every action, there is an equal and opposite reaction.

Whenever A exerts a force on B, B exerts a force on A that’s equal in size and opposite in direction.

All forces come in pairs.
Example of Newton’s Third Law:

Cookies push on hand: $F = 1$ pound, downward.
Hand pushes on cookies: $F = 1$ pound, upward.

Remove hand!

Earth pulls on cookies: $F = 1$ pound, downward.
Cookies pull on earth: $F = 1$ pound, upward.
THIRD Law states:

force on Earth = force on cookies

SECOND Law states:

acceleration = force divided by mass

Mass of Earth = $10^{25} \times$ mass of cookies

Therefore, acceleration of cookies = $10^{25} \times$ acceleration of Earth.

(Cookies reach a high speed while the Earth hardly budges.)
But…why do the cookies and the Earth exert a force on each other?

Newton’s Law of Gravity states that gravity is an attractive force acting between ALL pairs of massive objects.

Gravity depends on:

(1) MASSES of the two objects,

(2) DISTANCES between the objects.
Newton’s Law of Gravity:

The gravitational force between two objects

\[ F = G \left( \frac{Mm}{r^2} \right) \]

F = gravitational force
M = mass of one object
m = mass of the second object
r = distance between centers of objects
G = “universal constant of gravitation”
Gravitational force varies directly with mass and inversely with square of distance.

Double the distance between objects: Force 1/4 as large.

Triple the distance between objects: Force 1/9 as large.

\[ G = 6.7 \times 10^{-11} \text{ Newtons m}^2 / \text{kg}^2 \] (very small!)
Example: What is gravitational force between Earth and cookies?

\[ F = G \left( \frac{Mm}{r^2} \right) \]

- \( M \) = mass of Earth = \( 6.0 \times 10^{24} \) kg
- \( m \) = mass of cookies = 0.454 kg
- \( r \) = radius of Earth = \( 6.4 \times 10^6 \) m
- \( G \) = \( 6.7 \times 10^{-11} \) Newtons m\(^2\)/kg\(^2\)

Let's plug the numbers in:

\[ F = 4.4 \] Newtons = 1 pound
Example Encore: What is acceleration of cookies?

Second Law of Motion:

\[ a = \frac{F}{m} \]

Law of Gravity:

\[ F = G \left( \frac{Mm}{r^2} \right) \]

Therefore...

\[ a = G \left( \frac{Mm}{r^2} \right) \frac{1}{m} = \frac{GM}{r^2} = 9.8 \text{ m/sec}^2 \]

Independent of mass of cookies!!
Newton’s question: can GRAVITY be the force keeping the Moon in its orbit?

Newton’s approximation: Moon is on a circular orbit.

Even if its orbit were perfectly circular, the Moon would still be accelerated.
The Moon’s orbital speed:

radius of orbit: \( r = 3.8 \times 10^8 \text{ m} \)
circumference of orbit: \( 2\pi r = 2.4 \times 10^9 \text{ m} \)
orbital period: \( P = 27.3 \text{ days} = 2.4 \times 10^6 \text{ sec} \)

orbital speed:
\[
v = \frac{(2\pi r)}{P} = 10^3 \text{ m/sec} = 1 \text{ km/sec!}
\]
Acceleration required to keep Moon on a circular orbit

The required acceleration is:

\[ a = \frac{v^2}{r} \]

\( v = \) orbital speed, \( r = \) orbital radius

For the Moon:

\[ v = 10^3 \text{ m/sec} \]
\[ r = 3.8 \times 10^8 \text{ m} \]

\[ a = \frac{v^2}{r} = \frac{(10^3 \text{ m})^2}{3.8 \times 10^8 \text{ m}} = 0.0026 \text{ m/sec}^2 \]
Acceleration provided by gravity

\[ a = \frac{GM}{r^2} \]

At the surface of the Earth \((r = \text{radius of Earth})\)

\[ a = 9.8 \text{ m/sec}^2 \]

At the orbit of the Moon \((r = 60 \times \text{radius of Earth})\)

\[ a = \frac{9.8 \text{ m/sec}^2}{(60)^2} = \frac{9.8 \text{ m/sec}^2}{3600} \]

\[ a = 0.0027 \text{ m/sec}^2 \]

(close enough, given rounding errors)
Bottom Line

If gravity goes as one over the square of the distance,
Then it provides the right acceleration to keep the Moon on its orbit ("to keep it falling").

Triumph for Newton!!
Newton modified and expanded Kepler’s Laws of Planetary Motion

Kepler’s First Law:

The orbits of the planets around the Sun are ellipses with the Sun at one focus.

Newton’s revision:

The orbits of any pair of objects are conic sections with the center of mass at one focus.
As the Earth pulls on Moon, Moon pulls on Earth.

Both Earth and Moon orbit the center of mass of the Earth-Moon system:

Center of mass = balance point: closer to more massive object.
Artificial satellites as envisaged by Isaac Newton:

To put an object into orbit, launch it sideways with a large enough speed.

How large is large enough?
The shape of the orbit depends on the speed of the satellite at launch:

Low speed = *closed* orbit, a circle or ellipse.
High speed = *open* orbit, a parabola or hyperbola.

Circles, ellipses, parabolas, and hyperbolas are called *conic sections*. 
To remain in a circular orbit just above the Earth’s surface, a satellite must have $v = 7.9 \text{ km/sec}$.

To attain an *open* orbit, a satellite must reach at least 11.2 km/sec.
Some extra math:

First cosmic velocity:

\[ \frac{mv^2}{r} = G \frac{Mm}{r^2} \implies v_{\text{1st}} = \sqrt{\frac{GM_E}{R_E}} \]

\( v_{\text{1st}} = 7.9 \text{ km/sec} \)

Second (escape) cosmic velocity:

\[ \frac{mv^2}{2} = G \frac{Mm}{r} \implies v_{\text{2nd}} = \sqrt{\frac{2GM_E}{R_E}} \]

\( v_{\text{2nd}} = 11.2 \text{ km/sec} = \sqrt{2} \times v_{\text{1st}} \)
Kepler’s Second Law:
A line from a planet to the Sun sweeps out equal areas in equal time intervals.

Newton’s revision:
Angular momentum is conserved.
The product of the orbital speed ($v$) and the distance from the center of mass ($r$) is constant:

$$v \times r = \text{const}$$

As $r$ increases, $v$ must decrease.
angular momentum = \( m \times v \times r \)
Kepler’s Third Law:

\[ P^2 = a^3 \]

P = orbital period (in years)
a = semimajor axis (in A.U.)

Newton's revision:

\[ P^2 = \left[ \frac{4\pi^2}{G(M + m)} \right] a^3 \]

P = orbital period (in seconds)
a = semimajor axis (in meters)
G = universal constant of gravitation
M = mass of one object (in kilograms)
m = mass of other object (in kilograms)
Some extra math:

Newton's revision at work:

\[ P = \sqrt{\left[ \frac{4\pi^2}{G(M + m)} \right] a^3} \]

For the Earth's orbit

\[ M = 2 \times 10^{30} \text{ kg (mass of the Sun)} \]
\[ a = 1.5 \times 10^{11} \text{ m (1 A.U.)} \]
\[ G = 6.67 \times 10^{-11} \text{ m}^3 / \text{kg sec}^2 \]

\[ \Rightarrow P = 3.15 \times 10^7 \text{ sec!} \]
Kepler’s third law applies only to objects orbiting the Sun.

Newton’s revision applies to all pairs of object orbiting each other.

\[ M + m = \frac{4\pi^2 a^3}{GP^2} \]

Newton’s revision can be used to find masses of distant objects (e.g., binary stars).
Kepler described how planets move; Newton explained why they move that way.

Kepler’s laws result naturally from Newton’s laws of motion and Newton’s law of gravity.

Kepler’s laws of planetary motion, as modified by Newton, are UNIVERSAL!
Tides are caused by the difference between the Moon’s gravitational force on different sides of the Earth.

Time between high tides  
= 12 hours, 25 min  
Time between moonrises 
= 24 hours, 50 min
The gravitational force between two objects *decreases* as the distance between them *increases*.

The Moon’s gravitational pull on an object will be 7% greater on the *closer* side of the Earth than on the *further* side.
If the Moon’s pull were constant, then Earth would be undistorted.

After subtracting average pull, Earth is stretched in Moon’s direction.

Result: TWO tidal bulges, on opposite sides of Earth.
Why do we notice tides at the seashore?

**Rock is stiff:** Tidal bulges in rock are only 0.3 m high.

**Water is fluid:** Tidal bulges in water are 1 meter high.

Water bulges rise above rock bulges.
The Sun also creates tides on Earth.

High tides are highest when Sun, Earth and Moon line up (called ‘spring tide’).

High tides are lowest when Sun, Earth & Moon are at right angles (‘neap tide’).
Tides forces are slowing the Earth’s rotation and enlarging the Moon’s orbit.

The ocean’s tidal bulges press down on the ocean floor.

Friction robs energy from Earth’s rotation and uses it to heat the ocean.

This process is known as “tidal breaking”. (Think of the tidal bulges as brake pads!)

The length of the day is increasing by 0.002 seconds per per century.
Moon creates tidal bulges on Earth:
   Earth creates **BIGGER** bulges on Moon.
The Moon has **already** undergone tidal braking.
The Moon’s rotation has slowed to the point where rotation period equals the orbital period. **This** is why the Moon always keeps the same side turned to us.

[Google Moon](http://moon.google.com/)
Friction between the tidal bulges and ocean floor drags the bulges in the direction of the Earth’s rotation.

Bulges lead Moon by about 10 degrees.

The leading bulge steadily tugs the Moon into a larger orbit.

The average Earth-Moon distance is increasing by 4 meters per century.
How do we know the distance is increasing?
Measure the distance to the moon with great accuracy and watch it change!

Shoot the moon with a laser and watch it bounce off
How do we know the distance is increasing?
Measure the distance to the moon with great accuracy and watch it change!

Shoot the moon with a laser and watch it bounce off
Few closing questions:

1) If the Sun was twice as massive, how long would be the year on Earth?

2) Is the center of mass of the Solar System inside or outside the Sun?

3) How long will it take for a day on Earth to double in duration?

4) How long will it take for the Moon to double its distance from Earth?