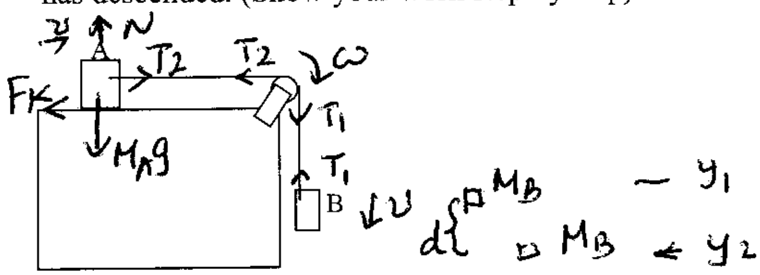
Physics 218-F2006
Mid term 3

Name: _

Section #: ______

Do not waste time on questions you are not sure of. Give ***clear answers*** and ***show all your work***. You **will not get full credit for a cluttered response or not fully showing your work.** **Writing down a number without showing your work will not earn full credit.** A sketch with variables defined, starting principles/equation appropriate for the problem and algebraic solutions with given variables will earn partial credit. Check the number of pages and be sure to put your section number and name.

1 The pulley in the figure has radius b and moment of inertia I. The rope does not slip over the pulley and the pulley spins on a frictionless axel. The coefficient of kinetic friction between the block A and the table is μ_k. The system is released from rest. Block A has mass M_A and block B has mass M_B. Use work energy principle to obtain the velocity of the block B as a function of the distance d it has descended. (Show your work step by step). Answer alone will earn only 5 points.



work energy (5) $K_1 + U_1 + W_{other} = K_2 + U_2$

(5) $0 + M_B g y_1 - \mu_k M_A g d = \frac{1}{2} M_B v^2 + \frac{1}{2} M_A v^2 + \frac{1}{2} I\omega^2 + M_B g y_2$

(5) $[M_B g(y_1 - y_2) - \mu_k M_A g d] = [(M_A + M_B)v^2 + I v^2/R^2]\frac{1}{2}$

$\therefore v^2[M_A + M_B + I/R^2] = 2(M_B g d - \mu_k M_A g d)$

(5) $v = \sqrt{\dfrac{2(M_B - \mu_k M_A) g d}{(M_A + M_B + I/R^2)}}$

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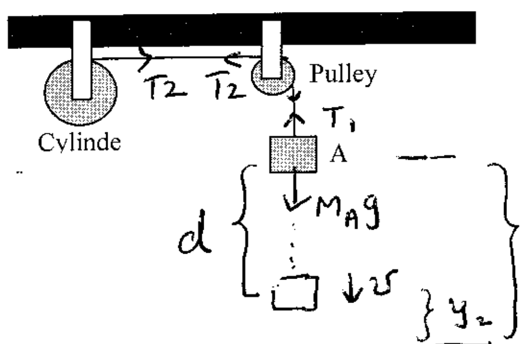
1. The cylinder (mass m) and the pulley (mass m) shown in the figure rotate without friction about stationary horizontal axes that pass through their centers. A massless rope is wrapped around the cylinder of radius 2R and passes over the pulley of radius R. Block A of mass M_A hangs from the end of the rope. Rope does not slip over the pulley or the cylinder when the block A is released. Obtain and expression to calculate the velocity of the block as a function of distance d it has descended (use work energy). Show your work step by step. Answer alone will earn only 5 pts.



(5)

Let us take the velocity as v and rotational speed of the pulley as ω_1

$\therefore v = R\omega_1$ --- (1)

angular velocity of the cylinder ω_2

$v = 2R\omega_2$ --- (2)

(5) $K_1 + U_1 + W_{other} = K_2 + U_2$

$$0 + M_A g y_1 + 0 = \frac{1}{2}M_A v^2 + \frac{1}{2}I_{pulley}\omega_1^2 + \frac{1}{2}I_{cyl}\omega_2^2 + M_A g y_2$$

(5) $\Rightarrow$

$$M_A g \underbrace{(y_1 - y_2)}_{d} = \frac{1}{2}M_A v^2 + \frac{1}{2}\left(\frac{mR^2}{2}\right)\left(\frac{v^2}{R^2}\right) + \frac{1}{2}\left[\frac{m(2R)^2}{2}\right]\cdot\frac{v^2}{(2R)^2}$$

$$= \frac{1}{2}v^2\left[M_A + \frac{1}{2}m + \frac{1}{2}m\right] = \frac{1}{2}v^2(M_A + m)$$

$$v^2 = \frac{2M_A g d}{M_A + m}$$

(5) $$v = \sqrt{\frac{2M_A g d}{(M_A + m)}}$$

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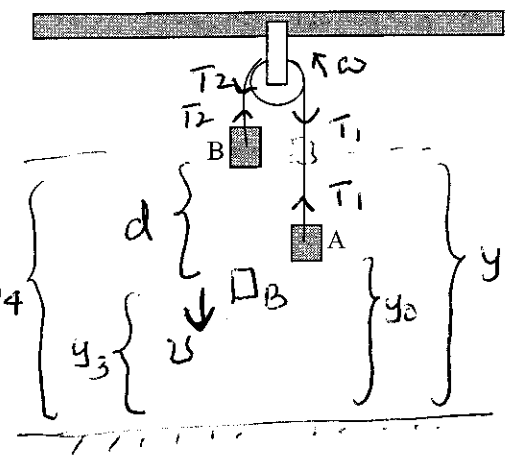
1. Blocks A and B with masses M_A and M_B are connected by a light rope as shown in the figure. Pulley (radius R and moment of inertia I) rotates on a frictionless axel. Mass M_B moves down when the system is released. Rope does not slip over the pulley. Obtain and expression to calculate the velocity of the masses as a function of distance d that the mass M_B descended (use work energy). Show your work step by step. Answer alone will earn only 5 pts.



velocity of the masses = v
angular velocity of the pulley at that time ω

$v = R\omega \Rightarrow \omega = v/R$ 5

5 $K_1 + U_1 + W_{other} = K_2 + U_2$

5
$$0 + M_B g y_4 + M_A g y_0 + 0 = \tfrac{1}{2} M_A v^2 + \tfrac{1}{2} M_B v^2 + M_A g y_1 + M_B g y_3 + \tfrac{1}{2} I \omega^2$$

$$M_B g \underbrace{(y_4 - y_3)}_{d} + M_A g \underbrace{(y_0 - y_1)}_{-d} = \tfrac{1}{2}(M_A + M_B)^2 + \tfrac{1}{2} I \frac{v^2}{R^2}$$

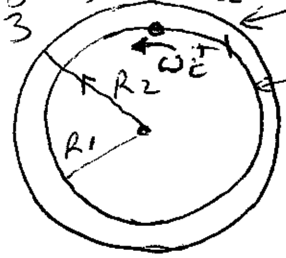
$$(M_B - M_A) g d = \tfrac{1}{2}\left(M_A + M_B + I/R^2\right) v^2$$

5
$$v = \sqrt{\frac{2(M_B - M_A) g d}{(M_A + M_B + I/R^2)}}$$

4

2. A horizontal disk with mass m and radius R can rotate without friction about a vertical axel through its center. A circular toy railroad track of radius 3R/4 and negligible mass is fixed symmetrically on the disk. A train (and batteries) of mass 4m/9 is placed on the track. When the switch is turned on using a remote control, train travels counterclockwise at a speed v with respect to the disk. a) Draw a picture looking down at the set up and indicate the angular velocity ω_d of the disk <u>with respect to ground</u> and the angular velocity ω_t of the train <u>with respect to ground</u>. b) What is the angular velocity ω of the train with respect to disk? c) What is ω_t in terms of ω_d and ω? (Remember part b) is relative angular velocity). d) Is the angular momentum conserved when the train is started? (Give reasons) e) Find ω_d for v=0.6m/s and R=0.48m. (Do not use number for parts a) to d))

(diagram) $\rightarrow \omega_d$
+3



+4b) $v = \frac{3}{4} R\omega$ +2 $\Rightarrow \omega = \frac{4v}{3R}$ +2 [only magnitude of ω's here]

+4 c) $\vec{\omega}_t - \vec{\omega}_d = \vec{\omega}$ +2 $\Rightarrow \vec{\omega}_t = \vec{\omega} + \vec{\omega}_d$ +2 or $\omega_t + \omega_d = \omega \Rightarrow \omega_t = \omega - \omega_d$

+4d) yes, No external torques in the direction of rotation.
$\therefore$ L conserves, before train starts L=0
after train starts

+4 e) L $I_t\omega_t - I_d\omega_d = 0$ +2

$\frac{4}{9} m R_1^2 \cdot \omega_t - \frac{1}{2} m R^2 \cdot \omega_d = 0$

$\frac{4}{9} m \cdot \frac{9}{16} R^2 (-\omega_d + \omega) - \frac{1}{2} m R^2 \omega_d = 0$

$\therefore (-\omega_d + \omega) - 2\omega_d = 0 \Rightarrow \underline{\frac{1}{3}\omega = \omega_d} = \frac{4 \times 0.6}{3 \ 0.48} = \frac{5}{9}$ rad/s +2

or using vector quantities.

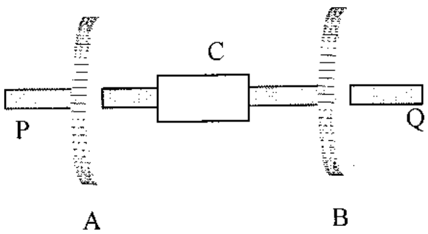
$I_t\vec{\omega}_t + I_d\vec{\omega}_d = 0 \Rightarrow \frac{4}{9} m \frac{9}{16} R^2 (\vec{\omega} + \vec{\omega}_d) + \frac{mR^2}{2}\omega_d = 0$

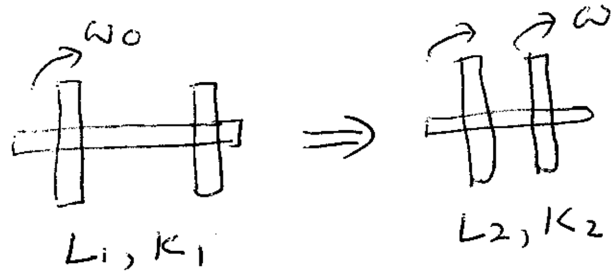
$\therefore \vec{\omega} + 3\vec{\omega}_d = 0 \Rightarrow \vec{\omega}_d = -\frac{1}{3}\vec{\omega} = -\frac{5}{9}$ rad/s ccw)

$= +\frac{5}{9}$ rad/s $\underline{CW}$

B

2 Two disks A and B, both having the same radii, are mounted on a shaft PQ going through their centers. Moments of inertia of the disks about the shaft are I_A and I_B, and I_A is four times I_B. The two disks can be connected or disconnected with the clutch C. With the disks disconnected, the disk B is brought up to an angular velocity ω_0 using a motor. Then the motor is removed and the two disks are connected using the clutch. It is found that 2400J is released when the two disks are connected. Ignore friction and the moment of inertia of the shaft and the clutch. Calculate the initial kinetic energy of the disk B. (show your work, a number alone will earn only 5 pts)





Let us take common angular velocity after connecting as ω

No external torques $\Rightarrow$ Conservation of L

$\Delta L = 0$ +5

$L_1 = I_B\omega_0 = L_2 = (I_A + I_B)\omega = 5 I_B \omega$ $\qquad I_A = 4 I_B$ +2

(doesn't have to be explicit, but they mus say it or use it)

$\therefore \omega = \frac{\omega_0}{5}$ +2 $\quad \cdots$ (1)

$K_1 = \frac{1}{2} I_B \omega_0^2, \qquad K_2 = \frac{1}{2}(I_A + I_B)\omega^2 = \frac{1}{2} 5 I_B \left(\frac{\omega_0}{5}\right)^2 = \frac{1}{2} I_B \omega_0^2 \left(\frac{1}{5}\right) \cdots$ (2)

$\Delta E = \omega$ (any variant) +5

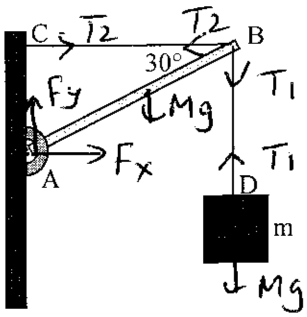
$\therefore K_1 - K_2 = \frac{1}{2} I_B \omega_0^2 \left(1 - \frac{1}{5}\right) = \frac{1}{2} I_B \omega_0^2 \frac{4}{5} = 2400\,J$

$\therefore \frac{1}{2} I_B \omega_0^2 = \underline{\underline{3000\,J}}$ +5

+1 for free

y

3. Show all the forces acting on the uniform strut AB of mass m and length $2l$. Find tension in each cable (CB and BD) and the magnitude and direction of the force exerted on the strut by the pivot at A.



(+3)

Take length of the rod $= 2l$

Reaction at A has F_x, F_y components

For Mass M:

$$\uparrow \Sigma F = 0 \Rightarrow T_1 - Mg = 0$$

$$T_1 = Mg \quad \cdots \text{(i)}$$

(+2)

$$A \circlearrowright \quad Mg\,l\cos 30 + \overset{T_1}{\cancel{Mg}}\,2l\cos 30 - T_2\,2l\sin 30 = 0$$

$$T_2\,2l\sin 30 = Mg\,l\cos 30 + T_1\,2l\cos 30 = 3Mg\,l\cos 30$$

$$T_2 = \frac{3Mg\cos 30}{2\sin 30} = \frac{3Mg\sqrt{3}}{2} = \frac{3\sqrt{3}Mg}{2} \quad \cdots \text{(2)}$$

(5)

For strut

$$\uparrow \Sigma F = 0 \Rightarrow F_y - Mg - T_1 = 0$$

$$\therefore F_y = Mg + T_1 = Mg + Mg = 2Mg$$

(4)

$$\rightarrow \Sigma F = 0 \Rightarrow F_x - T_2 = 0$$

$$F_x = T_2 = \frac{3\sqrt{3}}{2}Mg \quad (4)$$

$$\text{Reaction} \quad R = \sqrt{F_x^2 + F_y^2} = Mg\left(\frac{27}{4} + 4\right)^{1/2} = \frac{\sqrt{43}}{2}\,Mg$$

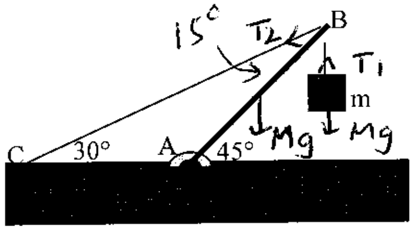
F_y, α, F_x

$$\tan\alpha = \frac{F_y}{F_x} = \frac{2Mg}{\frac{3\sqrt{3}}{2}Mg} = \frac{4}{3\sqrt{3}} \quad (2)$$

$$\alpha = \tan^{-1}\frac{4}{3\sqrt{3}}$$

B

3 Show all the forces acting on the uniform strut AB of mass m and length 2*l*. Find the tension in each cable and the magnitude and direction of the force exerted on the strut AB by the pivot.



+3

$AB = 2l$

A F_y F_x reaction at A

Mass M: $\sum F = 0 \uparrow \Rightarrow T_1 - Mg = 0$

$$T_1 = Mg \quad \ldots (1)$$

+2

A $\circlearrowright$ $\quad Mg\, l \cos 45 + Mg\, 2l \cos 45 - T_2\, 2l \sin 15 = 0$

$$\therefore\ 2T_2 \sin 15 = 3Mg \cos 45$$

$$T_2 = \frac{3Mg \cos 45}{2 \sin 15} \quad \ldots (2)$$

+5

$\uparrow \sum F = 0$ for strut

$$F_y - T_2 \cos 60 - Mg - T_1 = 0$$

$$\therefore F_y = T_2 \cos 60 + Mg + T_1 = \frac{3Mg \cos 45}{4 \sin 15} + Mg + Mg$$

$$= \frac{3}{4} Mg \frac{\cos 45}{\sin 15} + 2Mg = 4.049\, Mg$$

+4

$\rightarrow \sum F = 0 \Rightarrow -F_x + T_2 \cos 30 = 0$

$$\therefore F_x = T_2 \cos 30 = \frac{3\sqrt{3}\, Mg \cos 45}{4 \sin 15} = 3.549\, Mg$$

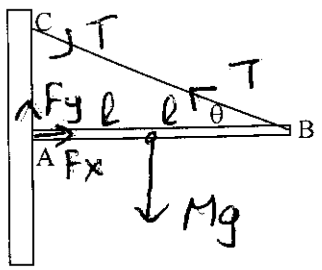
+4

$$R = \sqrt{F_x^2 + F_y^2} = (12.595 + 16.394)^{1/2} Mg = 5.38\, Mg$$

$$\tan \theta = \frac{F_y}{F_x} = \Rightarrow \theta = 48.8^\circ$$

+2

3 One end of a stick AB (length $2l$, *mass M*) is placed against a vertical wall. The other end is attached to the wall using a weightless cord CB that makes an angle θ with the stick. The coefficient of static friction between the end of the stick and the wall is μ_s. a) Find the tension in the cord, and the other forces acting on the stick. b) Obtain an expression for the maximum value of the angle if the stick to remain in equilibrium? (Use symbols) b) (5pts) Calculate the angle for μ_s=0.5



A) $Mg\cdot l - T\cdot 2l\sin\theta = 0$

$\therefore T = \dfrac{Mg}{2\sin\theta}$ --- (1)

(5)

$\uparrow \Sigma F = 0 \Rightarrow F_y - Mg + T\sin\theta = 0$

$\therefore F_y = Mg - T\sin\theta = Mg - \dfrac{Mg}{2} = \dfrac{Mg}{2}$ --- (2)

(5)

$\rightarrow \Sigma F = 0 \Rightarrow F_x - T\cos\theta = 0$

$\therefore F_x = T\cos\theta = \dfrac{Mg}{2\sin\theta}\cos\theta$ --- (3)

(5)

But $F_y = \mu F_x$

$\therefore \dfrac{Mg}{2} = \mu \dfrac{Mg}{2\sin\theta}\cos\theta$

$\therefore \mu = \tan\theta \le \mu_s$

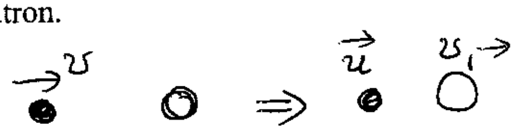
$\therefore \theta \le \tan^{-1}\mu_s$

(5)

for $\mu_s = 0.5$, $\theta = \tan^{-1} 0.5 = 26.56^\circ$

B Y

4 What is the difference between an elastic collision and an inelastic collision? (State, if any, what conserves and what does not conserve). A particle with mass m (neutron) and velocity v (assume moving from left to right in +x direction) makes a head-on elastic collision with another particle of mass M (nucleus) initially at rest. a) What is the initial kinetic energy K_I of the system? b) If the velocity of the neutron after the collision is u what is the velocity of the nucleus after the collision in terms of v and u? c) Is the momentum conserved in this collision? (Give reasons) d) Calculate the final velocity of the neutron and the nucleus. e) Obtain an expression for the kinetic energy lost by the neutron.



2

a) $K_1 = \frac{1}{2} m v^2$ ... (1)

b) $v - 0 = -(u - v_1)$ (relative velocity changes sign) 3

$\therefore v_1 = v + u$ ... (2) 2

c) yes, no external forces.

d) $\bar{p}$ conservation $\Rightarrow m v = m u + M v_1 = m u + M(v+u)$ -- (3) 2

$\therefore v(m - M) = u(m + M)$

$\therefore u = v(m-M)/(m+M)$ ... (4) 2

$\therefore v_1 = v + u = v + v\frac{(m-M)}{m+M} = \frac{2m}{m+M} v$ -- (5) 2

3

e) $K_1 - K_{n2} = \frac{1}{2} m v^2 - \frac{1}{2} m u^2 = \frac{1}{2} m v^2 - \frac{1}{2} m \left[\frac{v(m-M)}{(m+M)}\right]^2$

$= \frac{1}{2} m v^2 \left[1 - \frac{(m-M)^2}{(M+m)^2}\right] = \frac{1}{2} m v^2 \cdot \frac{4mM}{(m+M)^2}$ .. (6)

or elastic $\Rightarrow$ conserve energy

$\therefore \frac{1}{2} m v^2 = \frac{1}{2} m u^2 + \frac{1}{2} M v_1^2 \Rightarrow m(v^2 - u^2) = M v_1^2$ --- (7)

$m(v-u) = M v_1$ -- (8)

$\bar{p}$ conservation (3) $\Rightarrow$

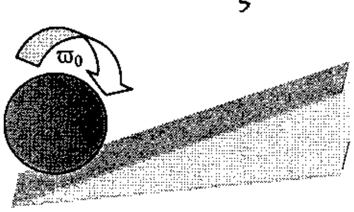
$v \neq u$, thus (7)/(8) $\rightarrow$

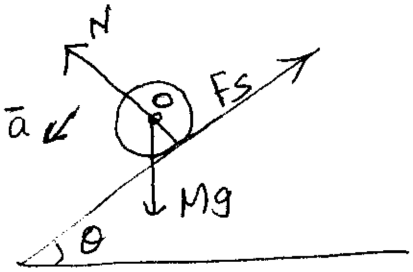
$(v+u) = v_1$ --- (9)

(9) is same as (2)

Teng W

5 A ball of mass M and radius R is rolled without slipping up a ramp that has an upward angle θ to the horizontal. Treat the ball as a uniform rigid sphere. a) Draw the freebody diagram. b) Give reasons for the direction of the force due to static friction. (Remember this is static friction and think of the consequences of the chosen direction to the angular velocity) c) Obtain an expression for the acceleration of the center of mass of the ball. d) Obtain the static frictional force and hence deduce the minimum coefficient of static friction needed to prevent slipping. Step by step derivation is required. $I = \frac{2}{5}MR^2$





a) 5.
b) 5.
c) 5.
d) 5

acceleration has to be downward, cannot speed up by itself.

b) If Fs pointing down the slope, it would increase rotational velocity. This cannot happen, violate energy conservation!

c) $\circlearrowleft_O \; \Sigma\tau = I\alpha \Rightarrow F_s R = I \cdot \alpha = \frac{2}{5}MR^2 \cdot \frac{a}{R}$

$$\therefore F_s = \frac{2}{5}Ma \quad \text{---} \quad (1)$$

$$\nearrow \Sigma F = Ma \Rightarrow Mg\sin\theta - F_s = Ma$$

$$Mg\sin\theta = Ma + F_s = Ma + \frac{2}{5}Ma = \frac{7}{5}Ma \quad \text{--} \quad (2)$$

$$\therefore a = \frac{5g\sin\theta}{7} \quad \text{---} \quad (3)$$

d) $F_s = \frac{2}{5}Ma = \frac{2}{5}M \cdot \frac{5}{7}g\sin\theta = \frac{2}{7}Mg\sin\theta \le \mu_s N$

$N = Mg\cos\theta$

$\therefore \frac{2}{7}Mg\sin\theta \le Mg\cos\theta \cdot \mu_s$

$\frac{2}{7}\tan\theta \le \mu_s$

$$\mu_s \geq \frac{2}{7}\tan\theta$$