

Physics 218-F2006

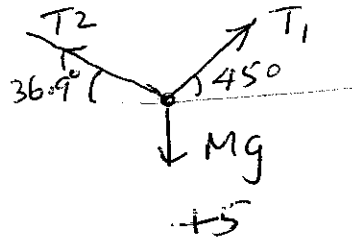
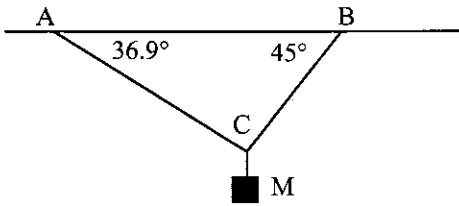
Mid term 2

Name: _____

Section #: _____

Do not waste time on questions you are not sure of. Give *clear answers* and *show all your work*. You will not get full credit for a cluttered response or not fully showing your work. Writing down a number without showing your work may not earn full credit. A sketch with variable defined, starting principles/equation appropriate for the problem and algebraic solutions with given variable will earn partial credit. Check the number of pages and be sure to put your section number and name.

1. A mass M is hanging at the joint of two pieces of ropes AC and BC which are attached to the ceiling as shown in the figure. Draw appropriate free body diagrams and find the tension in each rope. b) If ropes break when tension exceeds $100(\sqrt{2})g$ N, what is the maximum mass you can hang? c) Which rope breaks first? Ignore the mass of the ropes.



$\uparrow \Sigma F = (Ma \text{ or } 0)$
 $T_1 \sin 45 + T_2 \sin 36.9 - Mg = 0$

$\frac{T_1}{\sqrt{2}} + \frac{3}{5}T_2 = Mg \dots (1)$

$\rightarrow \Sigma F = (Ma \text{ or } 0)$
 $T_1 \cos 45 - T_2 \cos(36.9) = 0$

$\frac{T_1}{\sqrt{2}} - \frac{4}{5}T_2 = 0$

$\frac{T_1}{\sqrt{2}} = \frac{4}{5}T_2 \dots (2)$

(2) in (1) $\Rightarrow \frac{4}{5}T_2 + \frac{3}{5}T_2 = Mg$

$T_2 = \frac{5}{7}Mg \quad (.7142Mg) \quad +2$

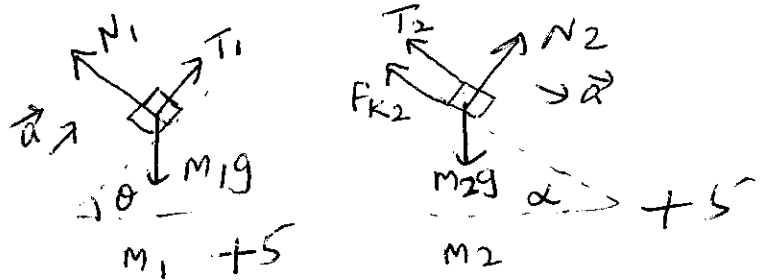
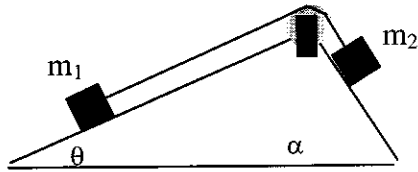
$T_1 = \frac{4\sqrt{2}}{5}T_2 = \frac{4\sqrt{2}}{7}Mg \quad (.808Mg) \quad +2$

b) $T_1 > T_2$ so BC breaks first, then

$T_1 = \frac{4\sqrt{2}}{7}Mg = 100\sqrt{2}g \Rightarrow M = 175Kg \quad +2$

c) BC breaks first since $T_1 > T_2$ +1 with correct logic

2. Two blocks of mass m_1 and m_2 are connected by a mass less rope that goes over a frictionless pulley which is attached to the vertex of a wedge as shown in the figure. Mass m_1 slides on a smooth surface and the coefficient of kinetic friction between mass m_2 and the surface it slides on is μ_k . Pulley is mass less and no air resistance. a) Draw appropriate free body diagrams. b) Assume m_2 is sliding down and obtain its acceleration, use symbols. c) Using above result deduce the acceleration if m_2 is sliding up.



Since pulley is frictionless massless, $T_2 = T_1$ + 2

$$m_2: \sum F = Ma \Rightarrow N_2 - m_2 g \cos \alpha = 0 \Rightarrow N_2 = m_2 g \cos \alpha$$

$$\therefore F_{k2} = \mu_k m_2 g \cos \alpha \quad \text{--- (1)} \quad + 2$$

$$\sum F = Ma \Rightarrow m_2 g \sin \alpha - T_2 - F_{k2} = m_2 a$$

$$m_2 g \sin \alpha - \mu_k m_2 g \cos \alpha - T_2 = m_2 a \quad \text{--- (2)} \quad + 2$$

$$m_1: \sum F = Ma \Rightarrow T_1 - m_1 g \sin \theta = m_1 a$$

$$T_2 - m_1 g \sin \theta = m_1 a \quad \text{--- (3)} \quad [\because T_2 = T_1] \quad + 2$$

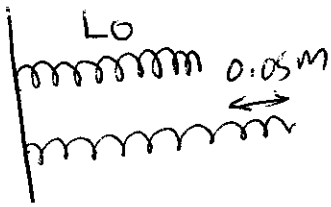
$$\text{(2) + (3)} \quad m_2 g (\sin \alpha - \mu_k \cos \alpha) - m_1 g \sin \theta = (m_1 + m_2) a$$

$$\therefore a = g \left[\frac{m_2 (\sin \alpha - \mu_k \cos \alpha) - m_1 \sin \theta}{(m_1 + m_2)} \right] \quad \text{---} \quad + 2$$

If m_2 is sliding up, $m_1 g$ and $m_2 g$ components reverse sign, but frictional force always against motion and does not change sign.

$$\text{So} \quad a_2 = g \left[\frac{m_1 \sin \theta - m_2 (\sin \alpha + \mu_k \cos \alpha)}{m_1 + m_2} \right] \quad + 5$$

- 3 A force of 160N is required to stretch a spring 0.05m from its un-stretched length. a) What is the force required to stretch the spring further 0.05m. b) How much work is done to stretch the spring further in part a)? c) What is the work needed to compress the spring by 0.05m from relaxed position?



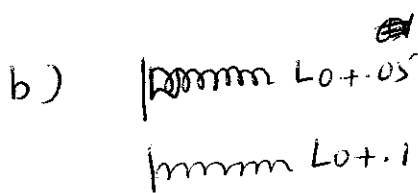
$$F_1 = kx_1$$

$$160 = k \times 0.05, \quad k = \frac{160 \text{ N/m}}{0.05} = 3200 \text{ N/m} \quad +4$$

$$\therefore F_2 = kx_2 = \underline{320 \text{ N}} \quad +4$$

OR

$$\left. \begin{array}{l} F_1 = kx_1 \\ F_2 = kx_2 \end{array} \right\} \Rightarrow F_2 = F_1 \frac{x_2}{x_1} = 160 \times \frac{0.1}{0.05} = \underline{320 \text{ N}}$$



$$W_1 = \frac{1}{2} kx_1^2 = \frac{k}{2} (0.05)^2$$

$$W_2 = \frac{1}{2} kx_2^2 = \frac{k}{2} (0.1)^2$$

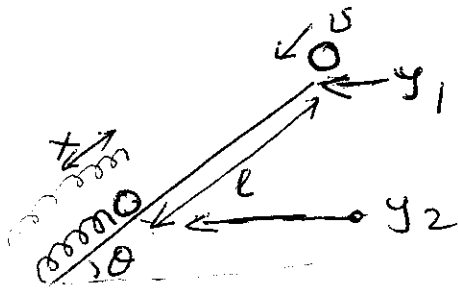
$$\therefore \text{work needed} = W_2 - W_1 = \frac{k}{2} [(0.1)^2 - (0.05)^2] =$$

$$= \frac{3200}{2} \times [0.15][0.05] = \underline{12 \text{ J}} \quad +6$$

c) Compression is same as stretching

$$W = \frac{1}{2} kx^2 = \frac{1}{2} 3200 \times (0.05)^2 = \underline{4 \text{ J}} \quad +6$$

- 4 A crate delivery ramp has a slope of $\theta=30^\circ$ down word from a horizontal conveyer belt. A crate moving with a velocity of $(4.9/2)\text{m/s}$ horizontally drops down the ramp which has a coefficient of (both kinetic and static) friction $\mu=1/\sqrt{3}$. A spring with spring constant k is fixed at the bottom of the ramp so that the crate compresses the spring by a distance x before it stops. Total distance traveled by the crate (down the ramp) before it stops is l . Once stopped the crate must not go up the ramp. Calculate x . b) Calculate the force constant of the spring to satisfy this if the crate is 1500N ? (Draw figures needed before proceeding with the problem, use work energy theorem).



$$y_1 - y_2 = l \sin 30 = l/2 \quad \text{--- (1) +2}$$



$$F_k = \mu_k Mg \cos \theta = \frac{1}{\sqrt{3}} Mg \times \frac{\sqrt{3}}{2} = \frac{Mg}{2} \quad \text{--- (2) +2}$$

$$K_1 + U_1 + W_{\theta} = K_2 + U_2$$

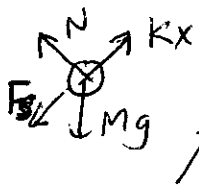
$$\frac{1}{2} m v^2 + Mg y_1 + F_k \cdot l = Mg y_2 + \frac{1}{2} k x^2 \quad +5$$

$$\frac{1}{2} m v^2 + Mg \underbrace{(y_1 - y_2)}_{l/2} - \frac{Mg l}{2} = \frac{1}{2} k x^2$$

$$\frac{1}{2} m v^2 + Mg l/2 - \frac{Mg l}{2} = \frac{1}{2} k x^2$$

$$\therefore k x^2 = m v^2 \quad \text{--- (1) 3}$$

Once stopped should not go up



$$F_s = \mu_k Mg \cos \theta = \frac{Mg}{2}$$

$$\sum F = ma \Rightarrow Kx - F_s - Mg \sin \theta = 0$$

$$Kx - \frac{Mg}{2} - \frac{Mg}{2} = 0$$

$$Kx = Mg \quad \text{--- (2) +5}$$

$$(1)/(2) \Rightarrow x = \frac{v^2}{g} = \frac{(4.9)^2}{4 \times 9.8} = \frac{4.9}{8} \text{ m} = 0.51 \text{ m} \quad +3$$

$$Kx = Mg = 1500 \text{ N}$$

$$K \frac{v^2}{g} = 1500 \text{ N} \Rightarrow K = \frac{1500g}{v^2} \text{ N/m} \quad +5$$

$$= \frac{12000}{4.9} \text{ N/m}$$

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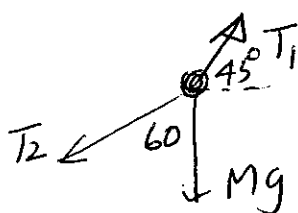
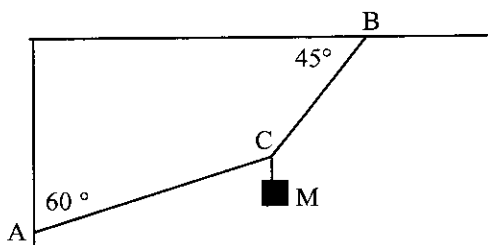
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1. A mass M is hanging at the joint of two pieces of ropes AC (attached to the wall) and BC (attached to the ceiling) as shown in the figure. Draw appropriate free body diagrams and find the tension in each rope. b) If ropes break when tension exceeds $100(\sqrt{3})g$ N, what is the maximum mass you can hang? c) Which rope breaks first? Ignore the mass of the ropes.



+5

$$\uparrow \Sigma F = 0 \Rightarrow T_1 \sin 45 - T_2 \cos 60 - Mg = 0$$

$$\frac{T_1}{\sqrt{2}} - \frac{T_2}{2} = Mg \quad \text{--- (1)}$$

+4

$$\rightarrow \Sigma F = 0 \Rightarrow T_1 \cos 45 - T_2 \sin 60 = 0$$

$$\frac{T_1}{\sqrt{2}} = T_2 \sin 60 = \frac{T_2 \sqrt{3}}{2} \quad \text{--- (2)}$$

+4

$$\text{(2) in (1)} \Rightarrow \frac{T_2(\sqrt{3}-1)}{2} = Mg$$

$$T_2 = \frac{2Mg}{\sqrt{3}-1} \approx 2.732Mg$$

+2

$$\text{in (2)} \quad T_1 = \frac{2mg \sqrt{3} \times \sqrt{2}}{(\sqrt{3}-1) \cdot 2} = \frac{\sqrt{6} Mg}{(\sqrt{3}-1)} \approx 3.346Mg$$

+2

b) $T_1 > T_2$ so BC breaks first

$$T_1 = 100\sqrt{3}g = \frac{\sqrt{3} \sqrt{2} Mg}{(\sqrt{3}-1)}$$

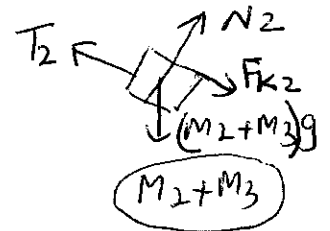
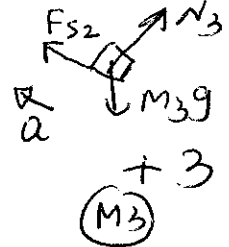
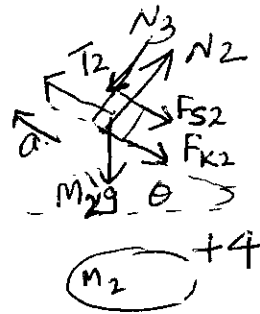
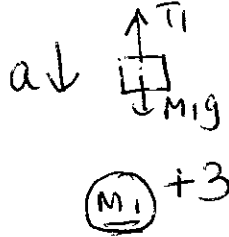
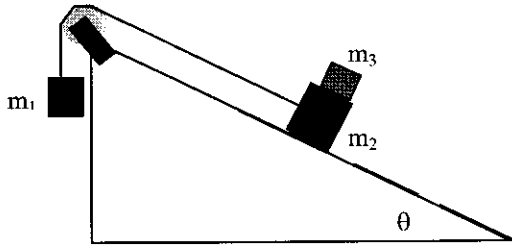
+2

$$M = \frac{100(\sqrt{3}-1)}{\sqrt{2}} \text{ kg} = \underline{\underline{51.2 \text{ kg}}}$$

c) $T_1 > T_2$ so BC breaks first

+1 with logic
not just statement

2. Three blocks with masses m_1 , m_2 , and m_3 are connected using a mass less rope which runs over a frictionless pulley as shown in the figure. Block m_2 slides up on a surface with coefficient of kinetic friction μ_k . Draw appropriate free body diagrams. Assume pulley is mass less and ignore air resistance. b) Obtain an expression for the acceleration a of mass m_1 . Given that the maximum value for coefficient of static friction between blocks m_2 and m_3 is μ_s , obtain the maximum value of a (in terms of μ_s and θ) if m_3 to stay together with mass m_2 .



Since pulley is frictionless & massless

$$T_1 = T_2 \quad \dots \textcircled{1} + 2$$

$\Sigma F = ma$ for

$$m_1: \downarrow M_1g - T_1 = M_1a \quad \dots \textcircled{2} + 2$$

$$(M_2+M_3): \nearrow N_2 - (M_2+M_3)g \cos\theta = 0$$

$$N_2 = (M_2+M_3)g \cos\theta$$

$$\therefore F_{k2} = \mu_k (M_1+M_2)g \cos\theta \quad \textcircled{3} + 2$$

$$\nearrow T_2 - (M_2+M_3)g \sin\theta - F_{k2} = (M_2+M_3)a \quad + 2$$

$$\Rightarrow T_1 - (M_2+M_3)g \sin\theta - (M_2+M_3)g \mu_k \cos\theta = (M_2+M_3)a \quad \textcircled{4}$$

$$\textcircled{4} + \textcircled{2} \quad [M_1 - (M_2+M_3)(\sin\theta + \mu_k \cos\theta)]g = (M_1+M_2+M_3)a$$

$$a = \frac{M_1 - (M_2+M_3)(\sin\theta + \mu_k \cos\theta)}{M_1+M_2+M_3} g \quad + 2$$

$$m_3: a_3 = a, \Sigma F = ma \Rightarrow$$

$$F_{s2} - m_3g \sin\theta = m_3a_3, \text{ but } F_{s3} \leq \mu_s N_3 = \mu_s m_3g \cos\theta$$

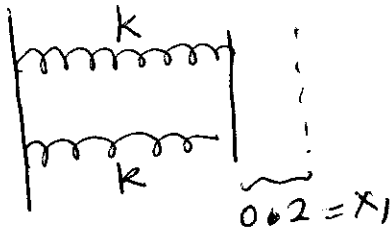
$$\therefore m_3a + m_3g \sin\theta = F_{s2} \leq \mu_s m_3g \cos\theta$$

$$\therefore a \leq \mu_s g \cos\theta - g \sin\theta$$

$$\leq \underline{\underline{g(\mu_s \cos\theta - \sin\theta)}}$$

+5 extra points

3. A leg press is made with two side by side horizontal springs and you push against them horizontally for your daily work out. You burn 80J when you press the platform 0.2m from their relaxed length. Ignore the mass of the spring. a) What is the magnitude of your force to hold the platform in this position? b) How many more joules are needed to press the platform 0.2m further from this position. c) What is the new force?

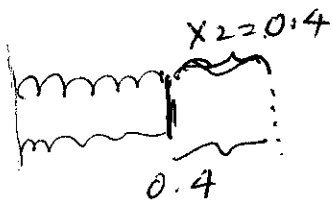


$$U_{el} = W = 2 \times \frac{1}{2} k x^2 = 80 \text{ J}$$

$$k = \frac{80}{(0.2)^2} \text{ N/m}$$

+ 4

$$\therefore F_1 = kx + kx = 2 \times \frac{80 \times 0.2}{(0.2)^2} = \underline{\underline{800 \text{ N}}} + 4$$



b) Now 0.2 to 0.4 compress.

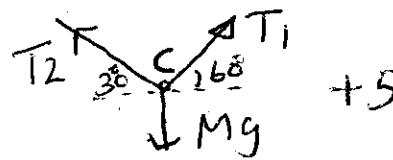
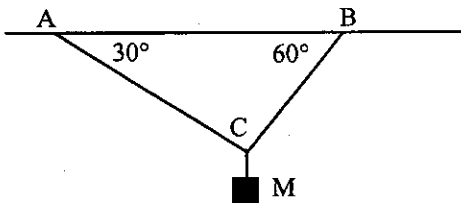
$$W = \left(\frac{1}{2} k x_2^2 - \frac{1}{2} k x_1^2 \right) 2 = k x_2^2 - k x_1^2 =$$

$$= \frac{80}{(0.2)^2} [(0.4)^2 - (0.2)^2] = 80 [4 - 1] = \underline{\underline{240 \text{ J}}} + 6$$

$$c) \text{ New force} = k x_2 + k x_2 = 2 \times \frac{80 \times 0.4}{(0.2)^2} = \underline{\underline{1600 \text{ N}}} + 6$$

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1. A mass M is hanging at the joint of two pieces of ropes AC and BC as shown in the figure. Draw appropriate free body diagrams and find the tension in each rope. b) If ropes break when tension exceeds $200(\sqrt{3})g$ N, what is the maximum mass you can hang? c) Which rope breaks first? Ignore the mass of the ropes.



$$\uparrow \Sigma F = 0 \Rightarrow T_2 \sin 30 + T_1 \sin 60 = Mg = 0 \quad +4$$

$$\frac{T_2}{2} + T_1 \frac{\sqrt{3}}{2} = Mg$$

$$T_2 + \sqrt{3} T_1 = 2Mg \quad \text{--- (1)}$$

$$\rightarrow \Sigma F = 0 \Rightarrow T_1 \cos 60 = T_2 \cos 30 \quad +4$$

$$T_1 = \sqrt{3} T_2 \quad \text{--- (2)}$$

$$\text{(2) in (1)} \Rightarrow T_2 + 3T_2 = 2Mg \quad +2$$

$$\therefore T_2 = Mg/2$$

$$T_1 = \frac{\sqrt{3} Mg}{2} \quad +2$$

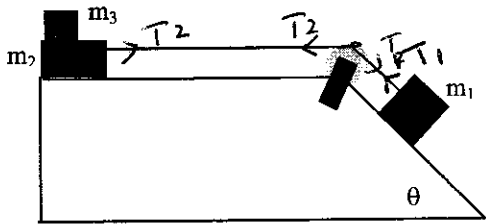
b) $T_1 > T_2$ Thus BC Breaks first $+1$

$$\therefore T_1 = \frac{\sqrt{3} Mg}{2} = 200\sqrt{3}g \text{ when breaks} \quad +1$$

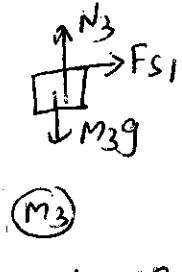
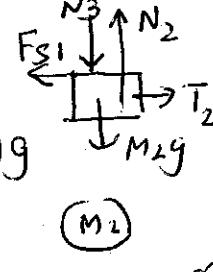
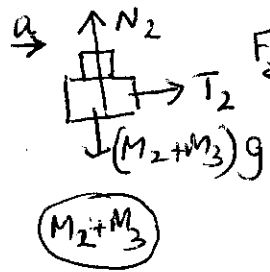
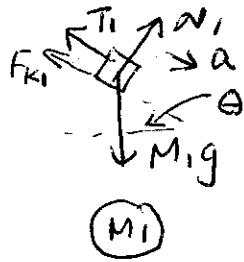
$$\therefore M = \underline{400 \text{ kg}}$$

c) BC breaks first since T_1 is larger $+1$ with correct logic

2. Three blocks with masses m_1 , m_2 , and m_3 are connected using a mass less rope which runs over a frictionless pulley as shown in the figure. Block m_3 slides on a frictionless surface and m_1 slides on a surface with coefficient of kinetic friction μ_k . Draw appropriate free body diagrams. b) Obtain an expression for the acceleration of mass m_1 . c) Given that the maximum value for coefficient of static friction between blocks m_2 and m_3 is same as μ_k , obtain an expression for the maximum value of μ_k (using θ and other given terms) if mass m_3 is to stay with mass m_2 . Pulley is mass less and ignore air resistance.



Three of the following four freebody diagrams are needed.



$$m_1: \sum F = Ma$$

$$N_1 \sin \theta = m_1 g \cos \theta = 0$$

$$N_1 = m_1 g \cos \theta$$

$$F_{k1} = \mu_k m_1 g \cos \theta \quad +2$$

$$m_1 g \sin \theta - \mu_k m_1 g \cos \theta - T_1 = m_1 a \quad +2 \quad (1)$$

$$(m_2 + m_3) \rightarrow T_2 = (m_2 + m_3) a \quad +2 \quad (2)$$

$T_1 = T_2$, since pulley is frictionless and massless

$$(2) \text{ in } (1) \Rightarrow m_1 g (\sin \theta - \mu_k \cos \theta) - (m_2 + m_3) a = m_1 a$$

$$\therefore a = \frac{m_1 g (\sin \theta - \mu_k \cos \theta)}{m_1 + m_2 + m_3} \quad +2$$

$$c) m_3: \sum F = Ma \Rightarrow F_{si} = m_3 a = m_3 a$$

$$\text{but } F_{si} \leq \mu_k N_3 = \mu_k m_3 g$$

$$\therefore m_3 a = F_{si} \leq \mu_k m_3 g$$

$$\therefore a \leq \mu_k g$$

$$m_1 g (\sin \theta - \mu_k \cos \theta) \leq \mu_k m_1 g$$

$$\frac{m_1 g (\sin \theta - \mu_k \cos \theta)}{(m_1 + m_2 + m_3)} \leq \mu_k g$$

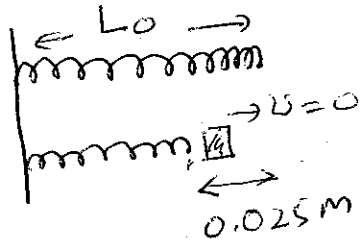
$$m_1 (\sin \theta - \mu_k \cos \theta) \leq \mu_k (m_1 + m_2 + m_3)$$

$$m_1 (\sin \theta) \leq \mu_k (m_1 + m_2 + m_3 + m_1 \cos \theta)$$

$$\frac{m_1 \sin \theta}{(m_1 + m_2 + m_3 + m_1 \cos \theta)} \leq \mu_k$$

+5 extra points.

3. A 4kg block of ice is placed against a horizontal spring that has force constant $k=200\text{N/m}$ and is compressed by 0.025m . Ignore the mass of the spring and the friction. a) Calculate the work done by the spring against the ice block when the spring is released while it expands to its natural length. b) What is the speed of the block when it leaves the spring? c) What would be the speed if the spring was compressed by 0.05m ?



$$\begin{aligned} \text{a) } W_{\text{spring}} &= -\Delta U = -(U_2 - U_1) = U_1 - U_2 \\ &= \frac{1}{2} kx^2 - 0 \\ &= \frac{1}{2} 200 \times (0.025)^2 = 0.0625 \text{ J} \end{aligned}$$

Since no friction

$$W_{\text{on ice}} = 0.0625 \text{ J} \quad +6$$

$$\text{b) } K_1 + U_1 = K_2 + U_2 \quad (\text{No friction})$$

$$0 + 0.0625 \text{ J} = \frac{1}{2} m v^2 + 0 = 2v^2$$

$$v^2 = \frac{0.0625}{2} \Rightarrow v = 0.176 \text{ m/s} \quad +8$$

c) compression doubled.

$$\frac{1}{2} kx^2 = 100 \times (0.05)^2 = (0.5)^2 \text{ J}$$

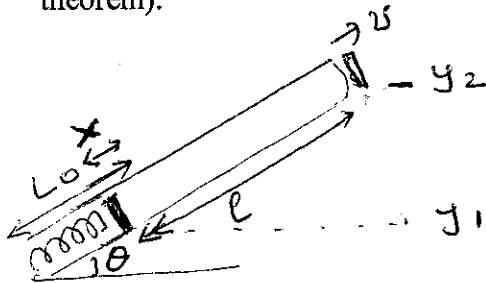
$$\frac{1}{2} m v^2 = (0.5)^2$$

$$2v^2 = (0.5)^2$$

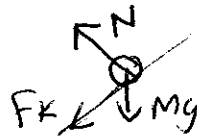
$$v = \underline{\underline{0.353 \text{ m/s}}}$$

+6

4. You are asked to design a spring that will shoot a mass m from a canon that has an upward angle θ , of which $\sin\theta = 5/13$ and $\cos\theta = 12/13$. The spring is compressed by a length x to shoot the mass and the mass travels a total distance of l through the cannon before it exits with a velocity v m/s. Inside of the canon is coated and the value of coefficient of kinetic friction is $1.5/12$. The maximum acceleration that the mass can be subjected to is $2g$. a) Obtain an expression that relates x , v^2 and l . b) Calculate the force constant k of the spring if $v = (4.9/2)$ m/s, $l = 2$ m and $m = 70$ kg. (use work energy theorem).



use work energy



$$F_k = \mu_k Mg \cos\theta$$

$$K_1 + U_1 + W_{other} = K_2 + U_2 \quad +5$$

$$0 + Mgy_1 + \frac{1}{2}Kx^2 - F_k l = \frac{1}{2}mv^2 + Mgy_2$$

$$\frac{1}{2}mv^2 = \frac{1}{2}Kx^2 - \mu_k Mg \cos\theta l + Mg(y_1 - y_2)$$

$$= \frac{1}{2}Kx^2 - Mg + \frac{1.5}{12} \times \frac{12}{13} l - Mg l \sin\theta$$

(since $y_2 > y_1$, $y_1 - y_2 < 0$)

$$= \frac{1}{2}Kx^2 - \frac{1.5Mgl}{13} - \frac{5}{13}Mgl$$

$$= \frac{1}{2}Kx^2 - \frac{1}{2}Mgl \quad \text{--- (1) } +5$$

Maximum force when spring is compressed
 $\Sigma F = Ma$

$$Kx - F_k - Mg \sin\theta = Ma$$

$$Kx - \mu_k Mg \cos\theta - Mg \sin\theta = ma$$

$$Kx - \frac{1.5Mg}{13} - \frac{5Mg}{13} = Ma = 2Mg$$

$$Kx = \frac{1}{2}Mg + M(2g) = 2.5Mg \quad \text{--- (2) } +5$$

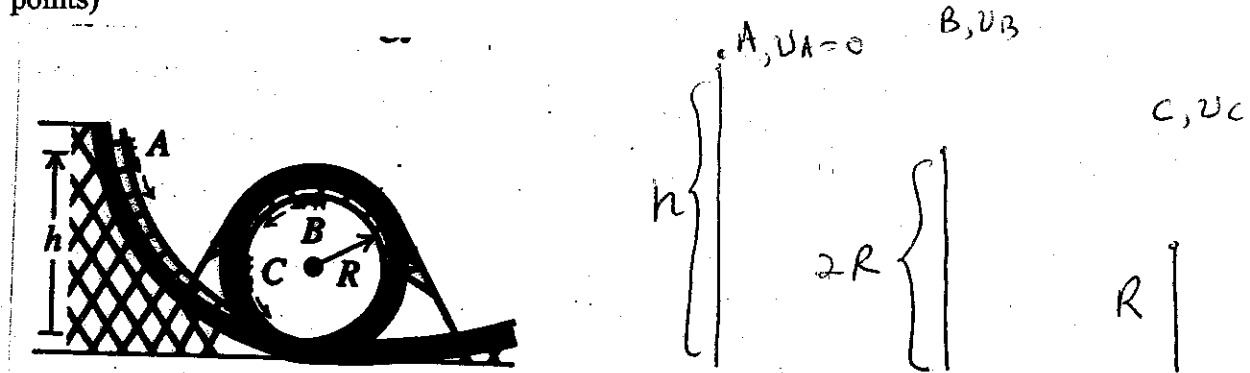
$$\text{(2) in (1)} \Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}2.5Mgx - \frac{1}{2}mgl$$

$$v^2 = g(2.5x - l) \quad \text{--- (3) } +5$$

$$\text{b) (3)} \Rightarrow x = \left(\frac{v^2}{g} + l\right) / 2.5 = \frac{20.9}{20} \text{ m}$$

$$\text{in (2)} \Rightarrow k = \frac{2.5Mg}{x} \approx \frac{5000}{2} \text{ N/m}$$

5. A car in an amusement park ride rolls without friction around a track shown in the figure. It starts from rest at point A at a height h above the bottom of the loop. Treat the car as a particle and ignore air resistance. a) Find the velocity v_B of the car at point B assuming it moves around the loop. b) What is the normal force at B? c) What is the minimum value of h such that the car moves around the loop without falling off the loop? d) What is the acceleration of the car at point C? (extra 5 points)



a) $K_1 + U_1 + W_{other} = K_2 + U_2$

$$0 + Mgh + 0 = \frac{1}{2}mv_B^2 + Mg(2R)$$

$$v_B^2 = \underline{\underline{2gh - 4gR = 2g(h - 2R)}}$$

b) F.B.D



$$\Sigma F = ma \Rightarrow \downarrow N + mg = \frac{mv_B^2}{R}$$

$$\therefore N = m\left(\frac{v_B^2}{R} - g\right) = m\left(\frac{2gh - 4gR}{R} - g\right)$$

$$= \underline{\underline{mg\left(\frac{2h}{R} - 5\right)}}$$

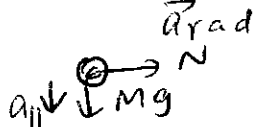
c) Not to fall $N > 0$ always except at B

$$\therefore N \geq 0 \Rightarrow mg\left(\frac{2h}{R} - 5\right) \geq 0$$

$$2h \geq 5R$$

$$\underline{\underline{h \geq 2.5R}}$$

d) at C F.B.D



$$a_{||} = \frac{Mg}{M} = g, \quad a_{\perp} =$$

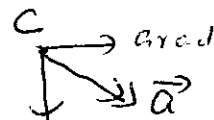
$N = \frac{mv_C^2}{R}$... NEED v_C

$K_1 + U_1 = K_2 + U_2$ gives

$$mgh + 0 = \frac{1}{2}v_C^2 m + mgR$$

$$\therefore v_C^2 = 2g(h - R)$$

$$a_{rad} = \frac{2g}{R}(h - R)$$



$a_{||}$

+ 5 extra points