This week we multiloop and RC circuits, and review for Thursday’s exam.

**Multiloop Circuits**

So far, you’ve learned to solve for currents and potential drops in relatively simple circuits, making combinations of resistors in series and in parallel. However, in some cases the circuits may be too complicated for this to be practical. For example, there can be more than one emf source in a circuit, which complicates things.

To understand how to solve more complicated “multiloop” circuits, we can make use of some equations called *Kirchhoff’s Rules*. But first we need to define a couple of new concepts:

1. **Junction**: A junction is any point on a circuit where *three or more* conductors meet.
2. **Loop**: A loop is any *closed* conducting path in a circuit.

Figure 1 shows examples of what are and aren’t junctions and loops in a multiloop circuit.

Kirchhoff’s rules take advantage of the concepts of loop and junction to define two rules that help with solving circuits:

1. **Kirchhoff’s Junction Rule**: The algebraic sum of all currents going into and out of a junction is zero:
   \[ \sum_{\text{junction}} I = 0. \]  
   (1)

2. **Kirchhoff’s Loop Rule**: The algebraic sum of all potential differences in any closed loop is zero:
   \[ \sum_{\text{closed loop}} V = 0. \]  
   (2)

Note that this applies to both potential increases (e.g. due to an emf source) and to potential drops (e.g. due to a resistor), with the following rules applying:

- Currents flowing into a junction are considered positive while currents flowing out of a junction are negative.
- The current flowing into a circuit element equals that flowing out of it.
• Crossing a battery from the $-$ to $+$ gives a positive potential of $+\mathcal{E}$. Crossing from the $+$ to $-$ terminal gives $-\mathcal{E}$.
• Crossing a resistor in the direction of current flow gives a negative potential $-IR$. Crossing a resistor against the current direction gives a positive potential $+IR$.

**Measurement Instruments**

There are two types of basic measurement instruments that can be used to probe circuits:

1. **Ammeters** measure the current flowing through a circuit branch. Ammeters are a “series device,” places in-line with the circuit branch in which you want to measure current. An ideal ammeter would have zero resistance to avoid altering the circuit it’s measuring. In reality this number is very low but not quite zero.
2. **Voltmeters** measure the potential difference, or voltage, between two points. Voltmeters are a “parallel device,” placed in parallel to the points you want to measure the potential drop across. An ideal voltmeter would have infinite resistance so that no current flows through it and it doesn’t alter the circuit.

Figure 2 shows example placements of a voltmeter and an ammeter in a circuit.

**Important points**

- **Ammeters** measure current (ideal: zero resistance).
- **Voltmeters** measure potential (ideal: infinite resistance).

Figure 2: Example placements of a voltmeter (labeled $V$) and an ammeter (labeled $A$) in a circuit.
RC Circuits

Until now, we’ve just dealt with circuits that are steady state, i.e. a constant current flowing through resistors due to an emf source. However, you can make combinations of resistors, emf sources, and capacitors where the current becomes time dependent.

Figure 3 shows an example of such an RC Circuit. There’s a switch in the circuit, initially open, that is closed at some time \( t_0 \). Here’s what happens:

• Initially, there is an open switch in the circuit, which means no current flows at all. The potential drop across the resistor is zero and the capacitor is uncharged.

• When you first close the switch, the charge on the capacitor is still zero. But current immediately begins flowing. At this instant it looks just like a resistor + emf, with current

\[
I_0 = \frac{E}{R}
\]

Next the capacitor begins charging up. During this time, the charge on the capacitor is increasing, decreasing the current flowing through the resistor. The equation describing this is:

\[
E = iR + \frac{q}{C}.
\]

This equation can be solved to get the current \( i \) and charge \( q \) as a function of time:

\[
i = I_0 e^{-t/(RC)}
\]

\[
q = Q_f \left[1 - e^{-t/(RC)}\right].
\]

What’s happening is that the current is decreasing exponentially, while the charge is increasing exponentially. Basically you’re trading current for charge on the capacitor.

• Eventually, the capacitor becomes fully charged (at \( t = \infty \)). From the equations above you can figure out what happens (remember: \( e^{-\infty} = 0 \)). The current drops to zero and the capacitor takes its full charge value (as if it were the only thing connected to the emf source):

\[
Q_f = EC.
\]

Figure 4 shows plots of the exponential decay of current and exponential increase of charge in such an RC circuit.

In Eqns. (5) and (6), the quantity \( RC \) is called the time constant, denoted by the symbol \( \tau \):

\[
\tau = RC.
\]
This is the characteristic time which describes how fast the current decreases or charge increases:

- **Small** $\tau$: the capacitor charges quickly.
- **Large** $\tau$: the capacitor charges slowly.

So far we’ve considered the case of *charging up* a capacitor that’s initially uncharged. However, there can also be a situation where the capacitor is *initially charged* to some value $Q_0$ and then *discharges*. Here the equations describing current and charge are both exponential decays:

\[
\begin{align*}
  i &= I_0 e^{-t/(RC)} \\
  q &= Q_0 e^{-t/(RC)}
\end{align*}
\]

This means that the current is initially at its maximum value $I_0$, at the moment the discharge and exponentially decays down to zero. Likewise, the charge on the capacitor also exponentially decays to zero from its maximum value of $Q_0$.

In the case where the circuit was charged by some battery with emf $\mathcal{E}$, the initial charge is

\[ Q_0 = \mathcal{E}C, \]

and the initial current is

\[ I_0 = \frac{\mathcal{E}}{R}. \]