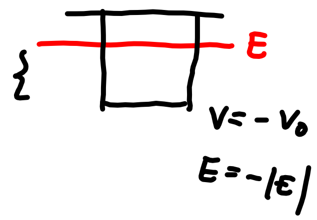
For solution a) $\alpha = k' \tan(\frac{1}{2}k'a)$



$$k' = \frac{1}{\hbar}\sqrt{2m(E-V)}$$

$$\alpha = \frac{1}{\hbar}\sqrt{2m|E|}^2 \qquad k' = \frac{1}{\hbar}\sqrt{2m(V_0-|E|)}$$

$$\hbar^2 k'^2 = 2mV_0 + \hbar^2\alpha^2$$

$$\alpha^2 = \frac{2mV_0}{\hbar^2} - k'^2 = k'^2 \tan(\tfrac{1}{2}k'a)$$

$$\Rightarrow \quad k'^2 \left[1+\tan^2(\tfrac{1}{2}k'a)\right] = \frac{2mV_0}{\hbar^2}$$

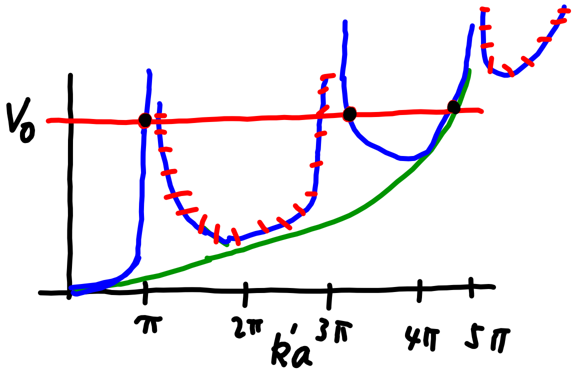
$$k'^2 \frac{1}{\cos^2(\frac{1}{2}k'a)}$$

$$V_0 = \frac{\hbar^2 k'^2}{2m}\frac{1}{\cos^2(\frac{1}{2}k'a)} \Rightarrow \text{solve for } k' \quad \text{Given } V_0 \text{ \& } a$$

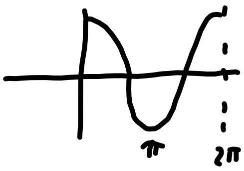
$$E = \frac{\hbar^2 k'^2}{2m} - V_0$$

The eigenvalue condition is

$$V_0 = \frac{(k'\hbar)^2}{2m}\frac{1}{\cos^2(\frac{1}{2}k'a)}$$



has poles at $\frac{1}{2}k'a = (n+\frac{1}{2})\pi$



at $k'a = (2n+1)\pi$

$\cos \to 1$ at $= m\pi$

$k'a = 2m\pi$

a) For any finite V_0, only a finite # of bound states

b) Always a bound state no matter how small is V_0.

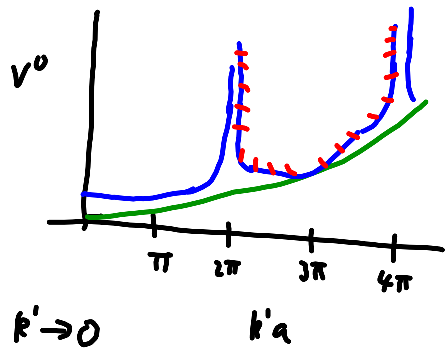
c) as $V_0 \to \infty$ $k'a = (2n+1)\pi$

$$E = \frac{\hbar^2 k'^2}{2m} = \frac{\hbar^2}{2m}\left[\frac{(2n+1)\pi}{a}\right]^2$$

For solution 2) odd solutions

$$V_0 = \frac{\hbar^2 k'^2}{2m}\frac{1}{\sin^2(\frac{1}{2}k'a)} \to$$

as $k' \to 0$



poles at $\frac{1}{2}k'a = n\pi$

$k'a = 2n\pi$

In contrast to case 1) No bound state if V^0 is too small.

The bound state of a delta-fct pot.

$$\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} \cos\xi - i\frac{\lambda}{2}\sin\xi & \eta\frac{i}{2}\sin\xi \\ -\eta\frac{i}{2}\sin\xi & \cos\xi + i\frac{\lambda}{2}\sin\xi \end{pmatrix}\begin{pmatrix} F \\ G \end{pmatrix}$$

$\xi = k'a \qquad \left(\frac{\hbar k'}{2m}\right)^2 = E - V \qquad r = \frac{k'}{k} \qquad \lambda = r + \frac{1}{r} \qquad \eta = \frac{1}{r} - \frac{1}{r}$

Consider the case $V \to \infty$ $a \to 0$ such that $Va \equiv v_0$

$$k' \to i\alpha \quad \frac{\hbar^2\alpha^2}{2m} = V - E \to \alpha \sim \sqrt{V}$$

$p = k\hbar$

$$\xi = i\alpha a = i\sqrt{V}a\frac{\sqrt{V}}{\sqrt{V}} = i\frac{aV}{\sqrt{V}} = i\frac{v_0}{\sqrt{V}} \to 0$$

$$\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 1 + i\frac{mv_0}{p\hbar} & i\frac{mv_0}{p\hbar} \\ -i\frac{mv_0}{p\hbar} & 1 - i\frac{mv_0}{p\hbar} \end{pmatrix}\begin{pmatrix} F \\ G \end{pmatrix}$$

For reflect + transmitted wave $G=0$

$\frac{p^2}{2m}=E$

$$A = \left(1+i\frac{mV_0}{\hbar p}\right)F \qquad \frac{F}{A} = \frac{1}{1+i\frac{mV_0}{\hbar p}} = S(\epsilon)$$

bound states are poles of $S(\epsilon)$

$\epsilon = -|\epsilon|$

$$p\hbar + i\,mV_0 = 0 \qquad p = \sqrt{2mE}$$

$$\cancel{i}\sqrt{2m|E|}\,\hbar + \cancel{i}\,mV_0 = 0 \qquad = i\sqrt{2m|E|} = i\alpha$$

$\hookrightarrow$ solution only if $\boxed{V_0 < 0}$

$$\sqrt{2m|E|} = \frac{m}{\hbar}|V_0|$$

$$E = -\frac{1}{2m}\frac{m^2}{\hbar^2}|V_0|^2 = -\frac{1}{2}\frac{m}{\hbar^2}V_0^2$$

$$\psi = e^{-\alpha|x|} \qquad \alpha = \frac{m}{\hbar}|V_0|$$

at $x>0$ $\quad \psi(x) = F e^{-\alpha x}$ $\qquad B = -i\frac{mV_0}{p\hbar}F = F$

$\quad\ \ x<0$ $\quad \psi(x) = B e^{\alpha x}$ $\qquad 1+i\frac{mV_0}{p\hbar} = 0$

A more direct solution of the delta fct pot.

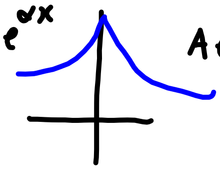
$$\int_{-\epsilon}^{\epsilon} dx\left(-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}\right)+\int_{-\epsilon}^{\epsilon} V_0\,\delta(x)\,\psi\,dx = E\int_{-\epsilon}^{\epsilon}\psi\,dx \qquad V_0<0$$

$\psi' = \frac{d\psi}{dx}$

since $\int\psi$ is continuous

$$-\frac{\hbar^2}{2m}\left[\psi'_+ - \psi'_-\right] + V_0\,\psi(0) = 0$$

$A e^{\alpha x}$ $A e^{-\alpha x}$



$$\boxed{\psi'_+ - \psi'_- = \frac{2m}{\hbar^2} V_0\,\psi(0)}$$

$\alpha\hbar = 2m|E|$

$$A(-\alpha) - A\alpha = \frac{2m}{\hbar^2} V_0\, A$$

$$-2\alpha = \frac{2m}{\hbar^2} V_0 \qquad E = -\frac{\hbar^2\alpha^2}{2m}$$

$$\alpha = \frac{m}{\hbar^2}|V_0| \qquad = -\frac{1}{2\cancel{m}}\,\frac{m^{\cancel{2}}\,V_0^2}{\hbar^2}$$