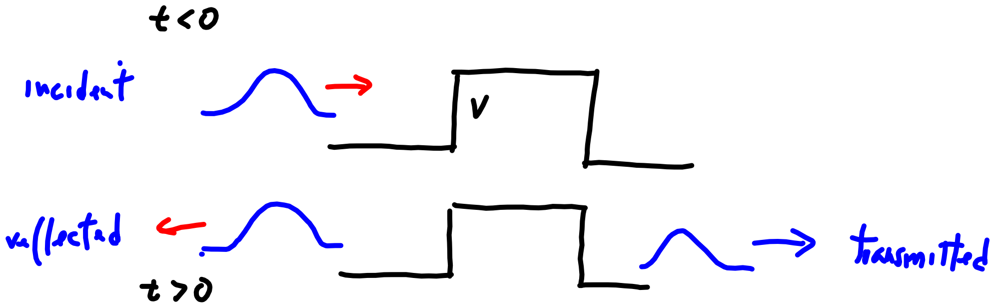
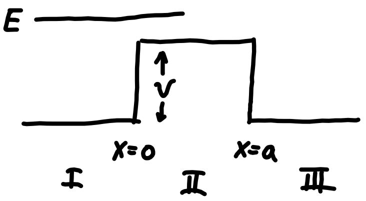
Potential problems in 1-D



Once the particle is detected, either as transmitted or reflected, then the wave function is collapsed at that pts.

Finite barrier/well



$k = \frac{p}{\hbar}$

$k\hbar = \sqrt{2mE}$

$E = \frac{p^2}{2m}$

$\hbar k' = \sqrt{2m(E-V)}$

$$\psi_I = Ae^{ikx} + Be^{-ikx}$$

$$\psi_{II} = Ce^{ik'x} + De^{-ik'x}$$

$$\psi_{III} = Fe^{ik(x-a)} + Ge^{-ik(x-a)}$$

a) match at $x=0$

$$A+B = C+D$$

$$\cancel{i}k(A-B) = \cancel{i}k'(C-D) \Rightarrow \begin{pmatrix} 1 & 1 \\ k & -k \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ k' & -k' \end{pmatrix}\begin{pmatrix} C \\ D \end{pmatrix}$$

b) match at $x=a$

$$\boxed{\xi = k'a}$$

$$Ce^{ik'a} + De^{-ik'a} = F+G$$

$$\cancel{i}k'(\underbrace{Ce^{ik'a}}_{\tilde{C}} - \underbrace{De^{-ik'a}}_{\tilde{D}}) = \cancel{i}k(F-G)$$

$$\begin{pmatrix} 1 & 1 \\ k' & -k' \end{pmatrix}\begin{pmatrix} \tilde{C} \\ \tilde{D} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ k & -k \end{pmatrix}\begin{pmatrix} F \\ G \end{pmatrix}$$

$$\begin{pmatrix} \tilde{C} \\ \tilde{D} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ k' & -k' \end{pmatrix}^{-1}\begin{pmatrix} 1 & 1 \\ k & -k \end{pmatrix}\begin{pmatrix} F \\ G \end{pmatrix}$$

$$\begin{pmatrix} A \\ B \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 1 \\ k & -k \end{pmatrix}^{-1}\begin{pmatrix} 1 & 1 \\ k' & -k' \end{pmatrix}}_{M}\begin{pmatrix} e^{-i\xi} & 0 \\ 0 & e^{+i\xi} \end{pmatrix}\underbrace{\begin{pmatrix} 1 & 1 \\ k' & -k' \end{pmatrix}^{-1}\begin{pmatrix} 1 & 1 \\ k & -k \end{pmatrix}}_{M^{-1}}\begin{pmatrix} F \\ G \end{pmatrix}$$

$$\begin{pmatrix} \tilde{C} \\ \tilde{D} \end{pmatrix} = \begin{pmatrix} e^{ik'a} & 0 \\ 0 & e^{-ik'a} \end{pmatrix}\begin{pmatrix} C \\ D \end{pmatrix}$$

$$\frac{1}{2}\begin{pmatrix} 1 & 1/k \\ 1 & -1/k \end{pmatrix} \qquad M = \frac{1}{2}\begin{pmatrix} 1+r & 1-r \\ 1-r & 1+r \end{pmatrix} \qquad M^{-1} = \frac{1}{2}\begin{pmatrix} 1+1/r & 1-1/r \\ 1-1/r & 1+1/r \end{pmatrix}$$

$$\boxed{r = \frac{k'}{k}}$$

$$\boxed{\lambda = \frac{1}{r} + r} \qquad \boxed{\eta = \frac{1}{r} - r}$$

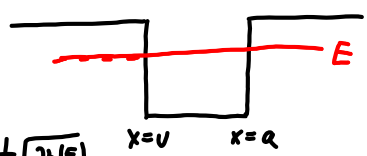
$$\boxed{\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} \cos(\xi) - i\frac{\lambda}{2}\sin(\xi) & \frac{i}{2}\sin(\xi)\eta \\ -\frac{i}{2}\sin(\xi)\eta & \cos(\xi) + i\frac{\lambda}{2}\sin(\xi) \end{pmatrix}\begin{pmatrix} F \\ G \end{pmatrix}}$$

det $= 1$

If $G = 0 \Rightarrow \frac{F}{A} = \frac{1}{\cos(\xi) - i\frac{\lambda}{2}\sin(\xi)} = S(\epsilon)$

Even if $A = 0$ $F \neq 0$ $\Rightarrow$ bound states are poles of $S(\epsilon)$

For $E < 0$



$V = -V_0$

$$k = \frac{1}{\hbar}\sqrt{2m(E)} = i\alpha \qquad \alpha = \frac{1}{\hbar}\sqrt{2m|E|}$$

$$r = \frac{k'}{k} = \frac{k'}{i\alpha}$$

$$\cos(\xi) - i\frac{\lambda}{2}\sin(\xi) = 0 \qquad \lambda = \frac{k'}{k} + \frac{k}{k'} = i\left(\frac{\alpha}{k'} - \frac{k'}{\alpha}\right)$$

$$\cos(\xi) + \frac{1}{2}\left(\frac{\alpha}{k'} - \frac{k'}{\alpha}\right)\sin(\xi) = 0 \qquad \xi = k'a$$

recall that

$$\cot(\xi) = \frac{1}{2}\left(\frac{k'}{\alpha} - \frac{\alpha}{k'}\right)$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$$\cot 2x = \frac{1-\tan^2 x}{2\tan x} = \frac{1}{2}(\cot x - \tan x)$$

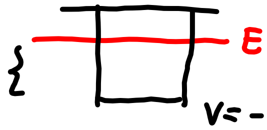
$$\boxed{\frac{1}{2}\left(\cot\tfrac{1}{2}\xi - \tan\tfrac{1}{2}\xi\right) = \frac{1}{2}\left(\frac{k'}{\alpha} - \frac{\alpha}{k'}\right)}$$

Two solutions: a) $\tan\frac{1}{2}\xi = \frac{\alpha}{k'}$ b) $\cot\frac{1}{2}\xi = -\frac{\alpha}{k'}$

For solution a) $\alpha = k' \tan(\frac{1}{2}k'a)$

$$k' = \frac{1}{\hbar}\sqrt{2m(E-V)}$$

$$\alpha = \frac{1}{\hbar}\sqrt{2m|E|}^{2} \qquad k' = \frac{1}{\hbar}\sqrt{2m(V_0-|E|)}$$



$V = -V_0$

$E = -|E|$

$$\hbar^2 k'^2 = 2mV_0 + \hbar^2\alpha^2$$

$$\alpha^2 = \frac{2mV_0}{\hbar^2} - k'^2 = k'^2 \tan(\tfrac{1}{2}k'a)$$

$$\Rightarrow \quad k'^2\left[1+\tan^2(\tfrac{1}{2}k'a)\right] = \frac{2mV_0}{\hbar^2}$$

$$k'^2 \frac{1}{\cos^2(\frac{1}{2}k'a)}$$

$$V_0 = \frac{\hbar^2 k'^2}{2m}\frac{1}{\cos^2(\frac{1}{2}k'a)} \Rightarrow \text{solve for } k' \quad \text{Given } V_0 \text{ \& } a$$

$$E = \frac{\hbar^2 k'^2}{2m} - V_0$$