

TRANSITION TO HOT QUARK MATTER IN RELATIVISTIC HEAVY-ION COLLISION*

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The nuclear matter-quark matter phase transition density is calculated as a function of temperature. The result suggests a transition to quark matter in heavy-ion collision at laboratory kinetic energies of a few GeV per nucleon. The transition may be inferred by observing a limiting temperature for the hadrons produced by the collision.

Our current understandings of strong interaction, as derived from studies of hadronic spectra [1] and lepton-nucleon scattering [2], are all based on the notion of weakly interacting quarks at short-distances [3] or at high momentum transfers [4,5]. An extrapolation of this idea implies that as matter is compressed to high density or heated to extreme temperature, a phase of quasi-free quark matter should appear [6]. If such a new state of matter does exist, then one likely way of finding it would be through high energy, central collision of heavy nuclei [7]. In a head-on, relativistic collision of heavy-ions, both high density (at least twice normal nuclear matter density and possibly four times as great if shock waves are formed [8]) and extreme temperature (≈ 100 MeV [9], see also below) are possible. In this note we shall calculate, on the basis of a modified MIT bag model for quark matter and a relativistic mean-field theory for nuclear matter, the density and temperature at which this transition occurs. We also propose a specific observation for inferring the existence of quark matter in heavy-ion collision distinct from those suggested in ref. [7].

We begin by considering the extent of heating possible in colliding nuclei. To describe hot nuclear matter, we adopt Walecka's finite temperature [10,11], relativistic mean-field theory [12,13]. In this theory, nucleons interact with each other via exchanges of a scalar and a vector meson. The parameters of the theory

are adjusted to give the correct binding energy and saturation density for nuclear matter at zero temperature. It represents the only fully relativistic, high temperature extrapolation of known nuclear matter properties currently available. To include the effect of pion production, we add to the mean-field theory, a gas of interacting pions. The latter's free energy, as calculated by Baym [14] on the basis of Weinberg's effective lagrangian, is given by

$$F = -V \int \frac{d^3k}{(2\pi)^3} \frac{k^2}{\omega_k} n_k + \frac{3}{8} V \left(\frac{m_\pi}{f_\pi} \int \frac{d^3k}{(2\pi)^3} \frac{n_k}{\omega_k} \right)^2, \quad (1)$$

where $n_k \equiv [\exp(\beta\omega_k) - 1]^{-1}$, $\beta = 1/T$, $\omega_k = (k^2 + m_\pi^2)^{1/2}$, and $f_\pi = 93$ MeV is the pion decay constant ($\hbar = c = k = 1$). No additional pion-nucleon interaction energy is included; presumably this has already been effectively taken into account when the parameters of the mean-field theory were adjusted to yield the exact nuclear matter binding energy and density. By identifying the energy per baryon in this calculation with the energy released per nucleon ϵ in the center-of-mass frame of two colliding identical nuclei, we obtain the "heating curves" in fig. 1. The laboratory kinetic energy per nucleon E_{lab} is related to ϵ by $E_{\text{lab}}/2M = (\epsilon/M)^2 - 1$, where M is the nucleon mass. For the case where the compressed baryon density is twice nuclear matter density, the results presented in fig. 1 are in qualitative agreement with an earlier estimate of ref. [9].

To determine a phase transition from nuclear to quark matter phenomenologically [15, 16], one needs

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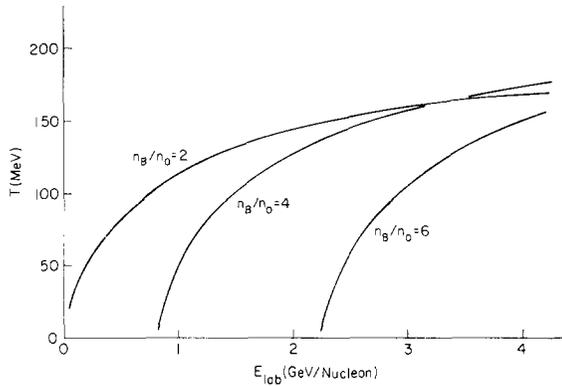


Fig. 1. Nuclear matter temperature for various compression ratios resulting from a head-on collision of two identical nuclei. It is assumed that one nucleus is at rest while the other impinge upon it with kinetic energy of E_{lab} per nucleon. n_B is the compressed baryon density. n_0 denotes the normal nuclear matter density = 0.17 fm^{-3} .

to know the free energy (or the pressure and the chemical potential) of both systems. For nuclear matter, this is readily obtainable from the mean-field theory with added pions. For quark matter, its free energy may be calculated on the basis of the MIT bag model [3, 17]. In this model, each hadron is viewed as a bag of quarks. A bag pressure B is introduced to account for the strong coupling effect of confinement. Quarks are then allowed to interact weakly via color gluons within the confine of the bag, i.e., inside hadrons. The parameters of the theory, B , $\alpha_c = g_c^2/4\pi$, z_0 (a zero-point energy), and the mass of the strange quark, are determined by fitting the masses of four known hadrons [3]. The u, d quarks are taken to be massless. According to this picture, at finite temperature, the “dissolving” of N nucleons in a volume V will result in N_i quarks of flavors i ($= u, d$) with the free energy density $f = F/V$ given by

$$f = B + \frac{1}{\pi^2} \left[-\frac{8}{45} \theta^4 + \sum_i \left(\frac{3}{4} \mu_{0i}^4 + \frac{1}{2} \mu_{0i}^2 \theta^2 - \frac{7}{60} \theta^4 \right) \right] + \frac{1}{\pi^2} \left(\frac{g_c}{\pi} \right)^2 \left[\frac{1}{6} \theta^4 + \frac{1}{8} \sum_i \left(\mu_{0i}^4 + 2 \mu_{0i}^2 \theta^2 + \frac{5}{9} \theta^4 \right) \right], \quad (2)$$

with $\theta \equiv \pi T$. The zero order chemical potential μ_{0i} is related to the quark density n_i by

$$n_i = N_i/V = \pi^{-2} (\mu_{0i}^3 + \mu_{0i} \theta^2). \quad (3)$$

The first three terms in eq. (2) are the free energies of the

bag, the color gluons, and the massless u, d quarks, respectively. The fourth term corresponds to the second order gluon–gluon interaction contribution. The present calculation of this term in the Feynman gauge agrees with the result obtained by Shuryak [18] in the Coulomb gauge. The fifth term is the second order quark–gluon interaction contribution. In the case of QED, this term has been previously evaluated by Akhiezer and Peletminskii [19], although they did not express their result in the above form. From eq. (2), one may determine the chemical potential and pressure via $\mu_i = (\partial F/\partial N_i)_{T,V}$ and $P = -f + \sum_i \mu_i n_i$. For the quark phase corresponding to nuclear matter, we also have $n_u = n_d = \frac{3}{2} n_B$ and $\mu_u = \mu_d$.

By Gibb’s criteria, nuclear matter is in phase equilibrium with quark matter when the temperature and the pressure of both phases are equal and the baryon chemical potential is related to those of the quarks by $\mu_B = 3\mu_u = 3\mu_d$. Thus at a given temperature, the baryon phase transition density may be determined by plotting the pressure of both phases against μ_B and locating the point of intersection. The result of this calculation, with the original MIT bag model values of $B^{1/4} = 145 \text{ MeV}$ and $\alpha_c = 2.2$, is shown in fig. 2. Although the form of the transition curve is possible, one must view this result with strong reservation. The coupling constant α_c in this case is actually too large for the perturbative calculation of (2) to be meaningful. The difficulty may be seen clearly by examining the energy density corresponding to (2) in the high temperature limit ($T \gg \mu_{0i}$). Since $E = F - T(\partial F/\partial T)$, we have from eq. (2),

$$E/V = B + \frac{37}{30} (\theta^4/\pi^2) \left[1 - \frac{55}{74} (g_c/\pi)^2 \right]. \quad (4)$$

For $\alpha_c \gtrsim 1$, the second order contribution is greater than the zero order result and the energy density is unbounded from below as $T \rightarrow \infty$. Both features are unacceptable. (The large bag-model coupling constant is also problematic for zero temperature quark matter calculation; see discussion in ref. [20].) In order not to abandon perturbation theory nor give up the bag model, we seek an alternative way of fitting hadronic masses that would yield a smaller value of α_c . For our present interest in the quark phase of nuclear matter, we may restrict attention to the nonstrange sector of the hadronic spectrum. There are thus four masses, m_p, m_π, m_Δ , and m_ω (the ρ and ω are degenerate in the bag model) need to be fitted with three parameters:

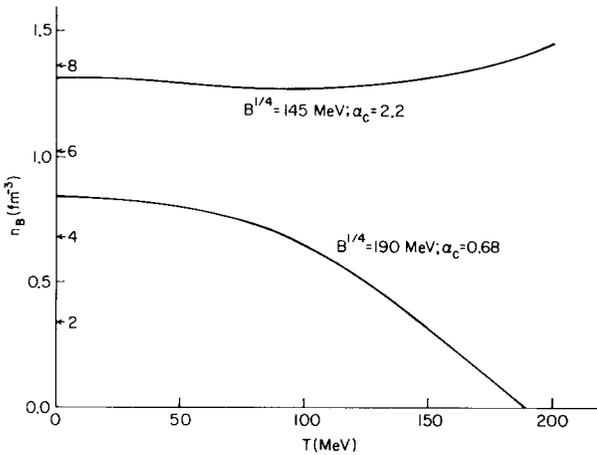


Fig. 2. The nuclear matter-quark matter phase transition density as a function of temperature. The upper curve is calculated using the original MIT bag values for $B^{1/4}$ and α_c . The lower curve uses modified values from fitting the pion mass exactly. The inner scale gives the baryon density in multiples of nuclear matter density.

B , α_c , and z_0 . The original bag model fixes these three by fitting masses of p , Δ , and ω . Their values are listed in table 1 along with the "predicted" pion mass and Regge trajectory slope α' . The result for α' is acceptable but the pion mass is off by a factor of two. Although there are good reasons (chiral symmetry) to expect an abnormally low mass for the pion, it might not be completely unsound phenomenologically to fit m_p and m_π exactly in order to compare the resulting quark matter with a hot nuclear matter that is consisted mainly of nucleons and pions. The modified set of bag parameters so determined by fitting m_p , m_π , and α' is given in the second row of table 1. The mass for ω is rather poor, but the smaller value of α_c is presently compatible with perturbation theory.

The resultant baryon transition density curve for $B^{1/4} = 190$ MeV and $\alpha_c = 0.68$ is also shown in fig. 2. In this case, even at zero temperature, the density re-

quired for the transition does not exceed five times nuclear matter density. For compression of twice nuclear matter density, a temperature of 147 MeV is needed. Referring back to fig. 1, this can be achieved with $E_{\text{lab}} \approx 2.3$ GeV/nucleon. If four times nuclear matter density is possible through shock wave formation, then the required temperature of 93 MeV can be met with $E_{\text{lab}} \approx 1.4$ GeV/nucleon. These threshold values of E_{lab} for quark matter production are, very surprisingly, in close agreement with those estimated by Chapline and Kerman [7] based on zero temperature considerations alone.

To investigate the experimental consequence of this transition, we consider the case of relativistic central collisions of heavy-ions with increasing E_{lab} . As suggested by recent experiments [21, 22], we assume that a blob of hot, thermalized nuclear matter is formed when two nuclei collide at high energy. As E_{lab} increases, the temperature of this hot mass of nuclear matter (as measured by the energy distribution of the emitted hadrons) would rise according to the heating curves of fig. 1. When E_{lab} exceeds the quark matter production threshold, hot and very dense quark matter would precipitate. The crucial observation here is that *the resultant quark matter must expand and cool back to the transition temperature before it can nucleate detectable hadrons*. Thus as E_{lab} increases indefinitely, the characteristic temperature of the emitted hadrons will show a limiting behavior. According to fig. 2, this limiting temperature does not exceed 190 MeV. More specifically, if one assumes that the transition takes place around twice nuclear matter density, then the limiting temperature is ≈ 150 MeV and sets in when $E_{\text{lab}} \gtrsim 2 - 3$ GeV/nucleon^{#1}. This idea of a limiting temperature in heavy-ion collision can be directly tested by experiment.

^{#1} Footnote: see next page.

Table 1

The first row gives the original determination of the bag parameters, the resulting hadron masses, and the Regge trajectory slope α' , as found in refs. [3, 17]. The second row gives the corresponding quantities with the set of modified bag parameters used in this paper. $B^{1/4}$ and all masses are in units of MeV. α' is in units of GeV^{-2} . The numbers inside parentheses denote experimental values.

$B^{1/4}$	α_c	z_0	$m_p(938)$	$m_\pi(139)$	$m_\Delta(1236)$	$m_\omega(783)$	$\alpha'(\approx 1)$
145	2.2	1.84	938	280	1233	783	0.88
190	0.68	3.4	941	137	1078	419	0.96

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^{#1} It should not escape attention that the formation of hot quark matter would provide a physical basis for a hydrodynamical description of particle production in heavy-ion collision similar to those previously proposed by Fermi, Pomeranchuk, and Landau. Moreover, by assuming a phase transition separating the quark phase from baryon matter, many phenomenological parameters introduced by these authors become calculable quantities. The nucleation temperature given above is but one example. Further discussions of this subject will be given in a separate publication.

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