

Name: \_\_\_\_\_,

PHYSICS 401 : SPRING SEMESTER 2018

**Project #9: Numerical Integrations**

**Reference Reading:** Sections 11:1-7.

**Downloads:** None needed.

- 1) a) Evaluate the integral analytically:

$$I = \int_{-1}^1 (a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5)dx.$$

We have shown in class that the 3-point Gaussian quadrature formula reproduces the exact answer  $I$ .

- a) What is the result,  $I_{3S}$ , obtained by three-point Simpson's rule in evaluating this integral? What is the error  $I_{3S} - I$ ? b) What is the result,  $I_{2G}$ , obtained by the two-point Gaussian rule in evaluating this integral? What is the error  $I_{2G} - I$ ?
- 2) a) Now compute the integral

$$I = \int_0^1 dx f(x) = \int_0^1 dx \frac{4}{1+x^2} \quad (1)$$

by use of a 3-point Simpson rule, a 2-point Gaussian, and a 3-point Gaussian rule. Compare your answer for  $I$  to  $\pi$ . (DO NOT symmetrize the integral to  $I = \frac{1}{2} \int_{-1}^1 dx f(x)$ , why?) b) Redo the integral as

$$I = \int_0^{1/2} dx \frac{4}{1+x^2} + \int_{1/2}^1 dx \frac{4}{1+x^2} \quad (2)$$

by apply each rule twice. What are your results  $I$  for the three rules?

- 3) Compute the integral (1) again but this time uses the simple Monte Carlo method. Use 10,000 sampling points and plot a)  $f(x_i)$  as a function of  $i$  from  $i = 1$  to  $i = 10,000$ , where  $x_i$  is uniformly distributed over  $[0,1]$ . Set the vertical scale of this graph from 1.9 to 4.1 so that you can see how  $f(x_i)$  fluctuates. b) Plot also the running average  $I_n = (1/n) \sum_{i=1}^n f(x_i)$  as a function of  $n$  and the line  $\pi$  (use different colors if possible). Hand in this graph.
- 4) Compute the integral (1) again but this time factor  $f(x) = h(x)p(x)$  and take  $p(x) = Ae^{-ax}$ . a) What must be  $A$  so that now  $\int_0^1 p(x)dx = 1$ ? b) Choose  $a$  so that  $f(x)/p(x)$  is as constant as possible and plot  $h(x_i)$  as a function of  $i$  from  $i = 1$  to  $i = 10,000$ , where  $x_i$  is now distributed according to  $p(x)$ . As in 2), plot both  $h(x_i)$  and its average  $I_n$ . *Be sure to use the same vertical scale as in 2) so that you can see the improvement.* Hand in this plot. What value of  $a$  would minimize the fluctuations of  $h(x_i)$ ? Hand in this plot.