Name: ____________________, ____________________

PHYSICS 401 : SPRING SEMESTER 2018

Project #5: Chaotic Maps

Reference Reading: Sections 3.4, 6.1-6.4, 6.7; App.6A

Downloads:

1) Download `ChaoticMapApp.java` from my website.

2) Go to http://www.eclipse.org/downloads/ and download the "Eclipse IDE for Java Developers", the shorter one. Install this in your Javawork directory. How to use Eclipse will be shown in class.

1. Henon's area preserving map is given by

$$\texttt{term} = \texttt{yold} - \texttt{xold} * \texttt{xold}, \quad \texttt{xnew} = \texttt{xold} * \texttt{c} - \texttt{term} * \texttt{s}, \quad \texttt{ynew} = \texttt{xold} * \texttt{s} + \texttt{term} * \texttt{c}$$

where $c = \cos(76.11°)$ and $s = \sin(76.11°)$. Examine `ChaoticMapApp.java` and see how this is implemented. Note that "c" and "s" are denoted as `rcos` and `rsin` in the Java file and are only evalued ONCE.

a) Use the following initial starting points: set $x_0 = 0.0$ always and set

$$y_0 = 0.1, 0.2, 0.3, 0.4, 0.441, 0.442, 0.5, 0.6, 0.65, 0.66, 0.665,$$
$$0.67, 0.68, 0.69, 0.71, 0.715, 0.72, 0.725, 0.73, 0.735$$

You need to iterate a few hundred to a few thousand times in each case, but plot ALL of them on a SINGLE graph. One way to do this is to store the above values in an array (read Section 3.4), and call them in another loop outside of the iteration loop. It maybe useful to return to Project1 and output all your result in a single text file, and then plot it using gnuplot. Hand in this graph. (Note that `nspeed` is initially set to 1 so that you can see which point is updated consecutively. For viewing on the screen, setting `nspeed=10-100` will greatly speed up the process.

b)First explore what happens from 0.441 to 0.442 using ChaoticMapApp. Then carefully output (x, y) ONLY if they are within the window $x = [0.2, 0.4]$ and $y = [0.4, 0.6]$. (Learn to use IF or While statements in Java to do this.) In the case of $y_0 = 0.442$, iterate up to million times to see the resulting structure. (DO NOT output a million points; check whether a points is inside the tiny window, only output it if it is.)

2. Explore the bifurcation behavior of the logistic map

$$x_{n+1} = 4rx_n(1 - x_n) \tag{1}$$

by plot the 'fixed points' as a function of r for $r = 0.75$ to $r = 1.0$. Divide the range [0.75,1.0] into 500 to 1000 (depending on your computer resources) discrete values of r_i. For each $r_i > 0.891$, iterate from $x_0 = 0.7$ for 600 times but only plot the last 300 values (all at the same r_i). For $r_i < 0.891$, iterate 50 times and just plot the next 30 iterations. If there are "drip" lines near the bifurcation points, increase the number of initial, discarded iterations.) Hand in this plot.

3. Now, explore the self-similarity of the bifurcation diagram by repeating the above calculation for $r = [0.862, 0.9196]$ and plot only those x-values in $x = [0.272, 0.728]$, *i.e.*, generate a plot with horizontal-axis from 0.862 to 0.9196 and vertical-axis from 0.272 to 0.728 . Describe what you see. (If you can, invert the top and the bottom.) Next, do it again for $r = [0.886, 0.898]$ and $x = [0.441, 0.590]$. And gain for $r = [0.8911, 0.8937]$ and $x = [0.4634, 0.5363]$. From the last two cases, give an estimate value of Feigenbaum's constants δ and α.

4. Iterate on the 'strange attractor' of Hennon (this is Problem 6.15a and 6.15b in Section 6.7) given by

$$\texttt{xnew} = \texttt{yold} + 1 - \texttt{a} * \texttt{xold} * \texttt{xold}$$
$$\texttt{ynew} = \texttt{b} * \texttt{xold}$$

with $a = 1.4$ and $b = 0.3$ by starting with an initial value x_0, y_0 anywhere within the unit circle. Plot the first 50-100 thousand points in the window $x = [-1.5, 1.5]$, $y = [-0.45, 0.45]$. Next, iterate 10^5 times and each time only plot those points that fall within the the window $x = [0.50, 0.75]$, $y = [0.15, 0.21]$. Do it again (possibly with more iterations if the image is too faint) for $x = [0.62, 0.64]$, $y = [0.185, 0.191]$. (Optional, do it again for $x = [0.6305, 0.6325]$, $y = [0.1889, 0.1895]$, with even more iterations, on the order of 10^6.) Hand in these 3 (or 4) plots.