Reduction from 2 → 1 body problem.

$H = p + e$

$$H = \frac{1}{2}\frac{p_1^2}{m_1} + \frac{1}{2}\frac{p_2^2}{m_2} + V(|\bar{r}_1 - \bar{r}_2|)$$

classically

$$H = \frac{1}{2M}\mathcal{P}_{cm}^2 + \frac{1}{2\mu}p^2 + V(r)$$

relative $\bar{r} = \bar{r}_1 - \bar{r}_2$

$$R = \frac{m_1\bar{r}_1 + m_2\bar{r}_2}{m_1 + m_2}$$

reduced mass

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = m_1 = m_e$$

$m_1 = m_e$

$m_2 = m_p$

$m_2 >> m_1$

Only do relative motion with $\mu \sim m_e$

Take $\mu \sim m_e = m$

$$\left(-\frac{\hbar^2}{2m}\nabla^2 - \frac{e^2}{r}\right)\psi = E\psi$$

$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ choose a, a length scale

$x = x^* a$
$y = y^* a$
$z = z^* a$

$\boxed{\frac{\hbar^2}{ma^2}}$ is an energy

$$-\frac{\hbar^2}{2ma^2}\nabla_*^2$$

all energy in units of $\frac{\hbar^2}{ma^2}$.

$$\left(-\frac{\hbar^2}{2ma^2}\nabla_*^2 - \frac{e^2}{r^* a}\right)\psi = E\psi$$

$\therefore E = E^* \frac{\hbar^2}{ma^2}$

$$\left(-\frac{1}{2}\nabla_*^2 - \frac{e^2}{\cancel{a}}\frac{ma^{\cancel{2}}}{\hbar^2}\frac{1}{r^*}\right)\psi = E^*\psi$$

choose a such that $\frac{e^2 ma}{\hbar^2} = 1 \implies a =$ Bohr's radius

$$\boxed{\left(-\frac{1}{2}\nabla_*^2 - \frac{1}{r^*}\right)\psi = E^*\psi}$$

$E^* = -\frac{1}{2}\frac{1}{n^2}$

$\frac{1}{2} E_h = 13.6\,eV$

$E_h = 2(13.6)\,eV$