Final Exam,

Next Friday, 12/7/2018

7:30 am – 9:30 am

Wednesday 11/28

my office hr

1:15 – 2:30 PM

4:00 – 4:30 PM

Next Monday, 12/3

no class

6 problems

↪ 2 similar to those in Exam 1

↪ 2 " " " " Exam 2

↪ 2 from stationary + time-dependent perturbation theory

Degenerate + near-Degenerate perturbation Theory

Rayleigh-Schrodinger perturbation

RS perturbation Theory

correction expanded $\sim \lambda \dfrac{\langle n|V|m\rangle}{\mathcal{E}_n - \mathcal{E}_m}$

$\sim$ ought to be small

would not work, or converges very slow if $\mathcal{E}_n \sim \mathcal{E}_m$

Absolutely fail if $\mathcal{E}_n = \mathcal{E}_m$ degenerate

unless we can arrange the eigenstates,

$H_0 |n_i\rangle = \mathcal{E}_n |n_i\rangle$ $\quad$ k-degenerate, $i = 1, k$

$$|n_\alpha\rangle = \sum_{i=1}^{k} C_{\alpha i} |n_i\rangle$$

choose $C_{\alpha i}$ so that $\boxed{\langle n_\alpha | V | n_\beta \rangle = 0 \, !}$ $\quad \alpha \neq \beta$

such that

$\langle n_\alpha | V | n_\alpha \rangle$ is non-vanishing

or

V diagonal in the α-basis.

simply diagonalize V in $\{n_i\}$

$\Rightarrow$ vanishing matrix elements that kill-off the vanishing denominator!

$$\langle n_i | V | n_j \rangle = \begin{pmatrix} \langle n_1|V|n_1\rangle & \langle n_1|V|n_2\rangle \\ \langle n_2|V|n_1\rangle & \cdots \end{pmatrix}$$

$$\det \begin{pmatrix} V_{11} - E & V_{12} & V_{13} & \cdots \\ V_{21} & V_{22} - E & \cdots & \end{pmatrix} = 0$$

solve for E_i

Ex : Stark Effect in hydrogen
↪ under an electric field

$\vec{E} = -\vec{\nabla}\phi$ for $\vec{E}$ const

$\phi = -\vec{E}\cdot\vec{r}$

$V = \underbrace{-e}_{\text{electron}}\phi = e\vec{E}\cdot\vec{r}$ take $\vec{E}$ in the z-axis

$$\boxed{V = eEz}$$

For the hydrogen atom $n=2$ 2s 2p $\begin{pmatrix} m=-1 \\ =0 \\ =1 \end{pmatrix}$

Note that : $[L_z, z] = 0$ $L_z = xP_y - yP_x$

$$0 = \langle l'm' | [L_z, z] | lm \rangle = m' \langle l'm' | z | lm \rangle - m \langle l'm' | z | lm \rangle$$
$$= (m'-m) \langle l'm' | z | lm \rangle$$

If $m' \neq m$ then $\langle l'm' | z | lm \rangle = 0$

The only non-vanishing matrix element is

$\langle 2s | z | 2p, m=0 \rangle = v$ finite matrix element

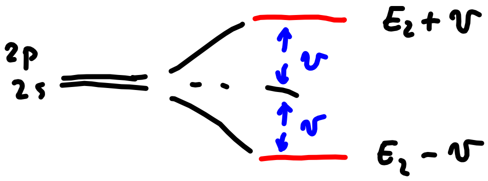
$$V = \begin{array}{c|cc} & 2s & 2p \\ \hline 2s & 0 & v \\ 2p & v & 0 \end{array} \Rightarrow \det\begin{pmatrix} -E & v \\ v & -E \end{pmatrix} = 0$$

$E^2 - v^2 = 0$ $E = \pm v$

for $E = v$ $\begin{pmatrix} -v & v \\ v & -v \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = 0 \Rightarrow \alpha = \beta$

↪ $\frac{1}{\sqrt{2}}\left(|2s\rangle + |2p\, m=0\rangle\right)$

$E = -v$ $\begin{pmatrix} v & v \\ v & v \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = 0 \Rightarrow \beta = -\alpha$ $\frac{1}{\sqrt{2}}\left(|2s\rangle - |2p\, m=0\rangle\right)$



Near-degenerate perturbation Theory

$|n\rangle$ + $|m\rangle$ are nearly degenerate

$$H = H_0 + V \qquad \text{submatrix } V_1$$

$$= H_0 + \begin{pmatrix} \langle n|V|n\rangle & \langle n|V|m\rangle \\ \langle m|V|n\rangle & \langle m|V|m\rangle \end{pmatrix} + (V - V_1)$$

Exactly diagonalize

$$H_0 + V_1 = \begin{pmatrix} E_n^{(1)} & \langle n|V|m\rangle \\ \langle m|V|n\rangle & E_m^{(1)} \end{pmatrix}$$

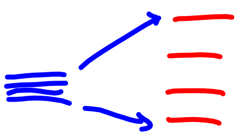
$$E_n^{(1)} = \varepsilon_n + \langle n|V|n\rangle$$

diagonalize this to obtain E'

$$\Rightarrow (E_n^{(1)} - E')(E_m^{(1)} - E') - |\langle m|V|n\rangle|^2 = 0$$

$$E'_{\alpha,\beta} = \frac{E_n^{(1)} + E_m^{(1)}}{2} \pm \sqrt{\left(\frac{E_n^{(1)} - E_m^{(1)}}{2}\right)^2 + |V_{mn}|^2}$$

For exact degeneracy $E_n^{(1)} = E_m^{(1)}$



$$E' = E_n^{(1)} \pm |V_{mn}|$$

Time-dependent perturbation Theory

At time $t < t_0 = 0$ $\quad i\hbar \frac{\partial}{\partial t}|\psi_t^0\rangle = H_0|\psi_t^0\rangle$ $\quad$ sudden turn-on

time $t > t_0$ $\quad$ turn on a time-dependent perturbation V_t

$$i\hbar \frac{\partial}{\partial t}|\psi_t\rangle = (H_0 + V_t)|\psi_t\rangle$$

Since the time-dependence is mostly given by H_0

$$|\psi_t\rangle = e^{-iH_0t/\hbar}|\psi(t)\rangle$$

$i\hbar\frac{\partial}{\partial t} \nearrow$

$$(\cancel{H_0} + V_t)\, e^{-iH_0t/\hbar}|\psi(t)\rangle = \cancel{H_0\, e^{-iH_0t/\hbar}|\psi(t)\rangle} + e^{-iH_0t/\hbar}\, i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle$$

$$i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = (\underbrace{e^{iH_0t/\hbar}\, V_t\, e^{-iH_0t/\hbar}})|\psi(t)\rangle$$

$$(*) \qquad \boxed{i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = V(t)|\psi(t)\rangle}$$

The state vector $|\psi_t\rangle$ evolves according to the Schrodinger picture with H_0. The potential evolves " " " Heisenberg picture with

↪ mixed - interaction representation.

The solution of the time-dependent Eq. (*) is given

$$|\psi(t)\rangle = T\left(e^{\frac{1}{i\hbar}\int_{t_0}^{t} V(t')\,dt'}\right)|\psi(t_0)\rangle$$

The first-order transition probability

At $t < t_0$ the system is in initial state $|0\rangle$ of H_0 with ε_0. (not necessary the g.s.)

After $t > t_0$, after we turn on the perturbation,

find the prob. that $|\psi_t\rangle$ is at the state $|n\rangle$ of H_0.

$$\langle n|\psi_t\rangle = \langle n|e^{-iH_0t/\hbar}|\psi(t)\rangle = e^{-i\varepsilon_n t/\hbar}\langle n|\psi(t)\rangle$$

$$\langle n|\psi(t)\rangle = \langle n|T(\ )|\psi(t_0)\rangle, \quad |\psi(t_0)\rangle = |0\rangle$$

Since $|\psi(t<t_0)\rangle = |0\rangle$

$$\langle n|\psi_t\rangle = e^{-i\varepsilon_n t/\hbar}\langle n|\left(1 + \frac{1}{i\hbar}\int_{t_0}^{t} V(t')dt' \cdots\right)|0\rangle$$

$$\langle n|\psi_t\rangle = \frac{1}{i\hbar}e^{-i\varepsilon_n t/\hbar}\int_{t_0}^{t}\langle n|V(t')|0\rangle dt'$$

$$\langle n|V(t')|0\rangle = \langle n|e^{iH_0t'/\hbar}V_{t'}e^{-iH_0t'/\hbar}|0\rangle$$

$$\boxed{|\langle n|\psi_t\rangle|^2 = \left|\frac{1}{i\hbar}\int_{t_0}^{t} e^{i(\varepsilon_n-\varepsilon_0)t'/\hbar}\langle n|V_{t'}|0\rangle dt'\right|^2 = P_{0\to n}(t)}$$