Stationary State Perturbation Theory chap. 11

$$H = H_0 + \lambda V$$

Known Hamiltonian — perturbation (changes) with parameter λ

$$H_0 |n\rangle = \varepsilon_n |n\rangle \qquad \langle n|n\rangle = 1$$

state energy

(keep track of perturbation order

$$H|N\rangle = E_N |N\rangle \qquad (1)$$

exact energy, exact wf

Assume that E_N & $|N\rangle$ are smoothly connected to H_0.

$H_0 \to H$

λ 0 → 1

Adiabatic Quantum Computing

$|N^{(0)}\rangle = |n\rangle$

$E_n^{(0)} = \varepsilon_n$

$$|N\rangle = |n\rangle + \lambda|N^{(1)}\rangle + \lambda^2|N^{(2)}\rangle + \dots \qquad (2)$$

$$E_n = \varepsilon_n + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)} + \dots \qquad (3)$$

normalization convention

$$\langle n|N\rangle = 1 \Rightarrow \langle N|N\rangle \neq 1$$

$$\| $$

$$1 + \lambda\langle n|N^{(1)}\rangle + \lambda^2\langle n|N^{(2)}\rangle + \dots \Rightarrow \boxed{\langle n|N^{(k)}\rangle = 0}$$

all corrections to $|n\rangle$ must be orthogonal to $|n\rangle$

Sub (2) + (3) into 1

$$(H_0+\lambda V)\left\{\sum_{k=0}^{\infty}\lambda^k|N^{(k)}\rangle\right\}$$

$$=\left(\sum_{i=0}^{\infty}\lambda^i E_N^{(i)}\right)\left(\sum_{\ell=0}^{\infty}\lambda^\ell|N^{(\ell)}\rangle\right)$$

$$H_0|n\rangle+\sum_{k=1}^{\infty}\lambda^k H_0|N^{(k)}\rangle + \underbrace{\sum_{k=0}^{\infty}\lambda^{k+1}V|N^{(k)}\rangle}_{\sum_{k=1}^{\infty}\lambda^k V|N^{(k-1)}\rangle}$$

$$=\sum_{i,\ell}\lambda^{(i+\ell)}E_N^{(i)}|N^{(\ell)}\rangle$$

$k=i+\ell$

$$\sum_{k=0}^{\infty}\left(\sum_{\ell=0}^{k}\lambda^k E_N^{(k-\ell)}|N^{(\ell)}\rangle\right)$$

For $k=1$

$$\cancel{H_0|n\rangle}+\lambda H_0|N^{(1)}\rangle+\lambda V|n\rangle$$

$$=\cancel{\mathcal{E}_n|n\rangle}+\lambda E_N^{(1)}|N^{(0)}\rangle+\lambda E_N^{(0)}|N^{(1)}\rangle$$

$$H_0|N^{(1)}\rangle+V|n\rangle=E_N^{(1)}|n\rangle+\mathcal{E}_n|N^{(1)}\rangle$$

dot with $\langle n|$

$$\langle n|V|n\rangle=E_N^{(1)}+0$$

$$E_N=\mathcal{E}_n+\lambda E_N^{(1)}=\mathcal{E}_n+\lambda\langle n|V|n\rangle$$

first order perturbation result