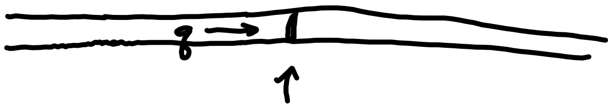
Electric Charge → Electric field → Electric flux → Electric potential → Electric flow → Electric circuit

$\vec{E}$

infinite conductor



electric current $I = \frac{\Delta q}{\Delta t} = \frac{\text{amount of charge}}{\text{unit time}}$

Const I ⇒ const velocity flow

for a const E ⇒ $F = qE$
$\quad\quad\quad\quad\quad\quad\quad ma = qE$

Const $\vec{E}$ → const acceleration ⇒ $a = \frac{qE}{m}$

$v = at = \frac{qE}{m} t$

However, with $\vec{E}$

$\vec{v_d}$

for $t = \tau$ ← average time between collision

$v = v_d = \frac{qE}{m} \tau$ drift velocity

under $\vec{E}$ + collision ⇒ const velocity flow
⇒ I

↓

potential V ⟶ current flow

for Given V, cause a current I of

$$I = \frac{V}{R}$$

Current → I

V ← pot diff or voltage

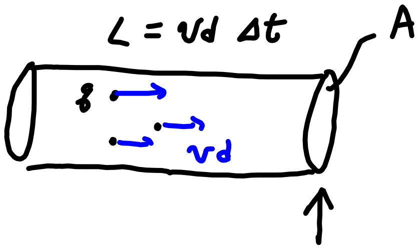
R ← resistance of the material

$\left[a = \frac{F}{m} \right]$

Ohm's Law.

Current flows in direct proportion to the applied voltage & inversely proportional to the resistance.

More details on Ohm's Law:



Let $n = \dfrac{\#\text{ of charges}}{\text{Volume}}$ = charge density

find current

$$I = \frac{\Delta q}{\Delta t} = \frac{q\, \text{Vol} \cdot n}{\Delta t} = \frac{(A v_d \cancel{\Delta t})\, n\, q}{\cancel{\Delta t}}$$

$$I = A v_d n q$$

$$= A \frac{qE\tau}{m} n q$$

$V = Ed$

$= EL$

$$I = \frac{Aq}{m} \frac{V}{L} \tau n q = \frac{AV}{L} \frac{q^2 \tau n}{m}$$

$$= \frac{Aq}{m} n \frac{V}{L} = \frac{A}{L} \frac{V}{\rho}$$

$\rho = \dfrac{m}{q^2 \tau n}$ ← resistivity of the material

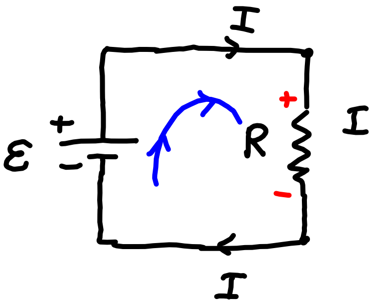
$$I = \frac{A}{L\rho} V = \frac{V}{R}$$ Ohm's Law

$$\boxed{R = \frac{L\rho}{A}}$$

Current flow is maintained by a "potential pump"

〃

battery = emf



$$\Delta V_{\varepsilon} + \Delta V_R = 0$$

$$\varepsilon - V = 0, \quad V = IR$$

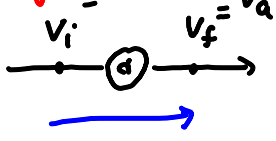
$$\varepsilon = IR \Rightarrow I = \frac{\varepsilon}{R}$$

1) + sign where the current enters the device

\- sign where it exists

pot diff

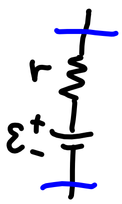
voltage diff

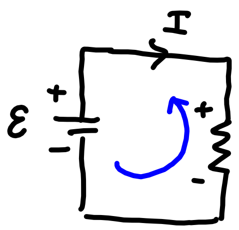


2) Kirchhoff Rule:

The sum of all pot. diff. across all devices must be zero.

choose a direction of calculation: $\Delta V = V_{after} - V_{before}$





$$\Delta \varepsilon + \Delta R = 0$$

$$-\varepsilon + IR = 0$$

$$I = \frac{\varepsilon}{R} \,!$$

Power dissipation in a resistor

$$P = \frac{\Delta W}{\Delta t} = \frac{\text{work done}}{\text{unit time}} = \frac{\Delta U}{\Delta t} = \frac{qV}{\Delta t} = IV$$

(pot.)

$$P = IV$$

since

$$= I^2 R$$

$$V = IR$$

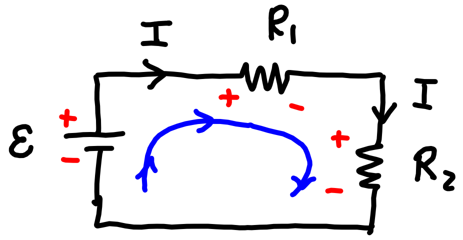
$$= \frac{V^2}{R}$$

$$\rightarrow I = \frac{V}{R}$$

either current → $I^2 R$

or the voltage → $\frac{V^2}{R}$

Apply Kirchhoff's rule



$$\Delta V_{\varepsilon} + \Delta V_{R_1} + \Delta V_{R_2} = 0$$

$$\varepsilon - IR_1 - IR_1 = 0$$

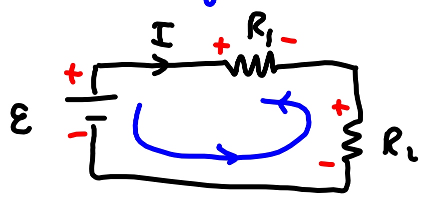
$$\varepsilon = I(R_1 + R_2)$$

$$I = \frac{\varepsilon}{R_1 + R_2} \quad \leftarrow R_{eq}$$

Resistors in series:

$$R_{eq} = R_1 + R_2 + R_3 \; ..$$

Same Current



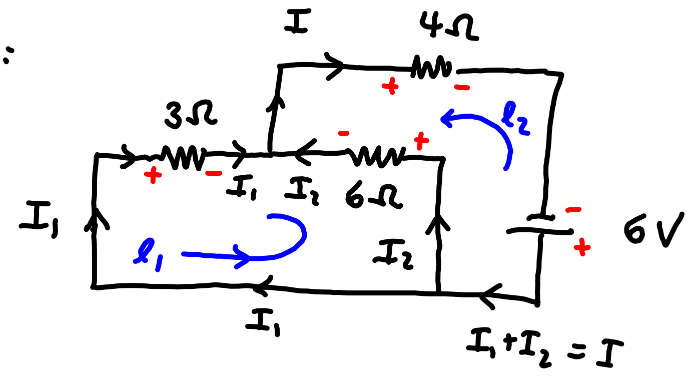
$$\Delta V_{\varepsilon} + \Delta V_{R_1} + \Delta V_{R_2} = 0$$

$$-\varepsilon + IR_2 + IR_1 = 0$$

$$I = \frac{\varepsilon}{R_1 + R_2}$$

Ex 3:

Find all currents flowing thru each device



ℓ_1 : $\Delta V_6 + \Delta V_3 = 0$

$-I_2 6 + I_1 3 = 0$

$3I_1 = 6I_2$

$$\boxed{I_1 = 2I_2}$$

$$\boxed{I_2 = \frac{6}{18} = \frac{1}{3}}$$

ℓ_2 : $-6 + 4(I_1 + I_2) + 6I_2 = 0$

$6I_2 + 4I_1 + 4I_2 = 6$

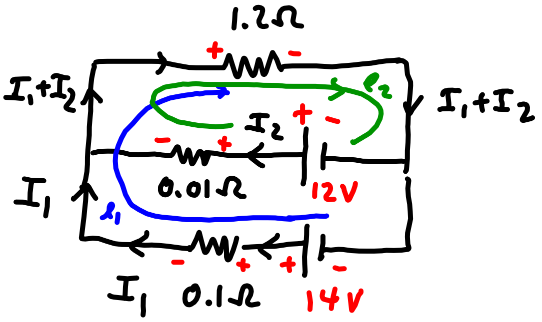
$10I_2 + 4I_1 = 6$

$10I_2 + 4(2I_2) = 6$

$18I_2 = 6$

Ex 4 : Two batteries

Find all current thru each resistors.



ℓ_1 : $14 - I_1\, 0.1 - (I_1+I_2)\,1.2 = 0$

ℓ_2 : $12 - I_2\, 0.01 - (I_1+I_2)\,1.2 = 0$

$$14 = 1.3\, I_1 + 1.2\, I_2$$
$$12 = 1.21\, I_2 + I_1\, 1.2$$

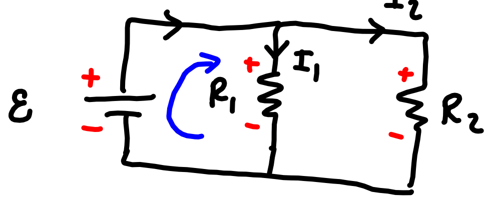
$$I_2 = \frac{12 - I_1\, 1.2}{1.21}$$

$$14 = 1.3\, I_1 + 1.2\left(\frac{12 - I_1\, 1.2}{1.21}\right)$$
$$= 1.3\, I_1 + \frac{1.2 \cdot 12}{1.21} - I_1 \frac{(1.2)^2}{1.21}$$

$I_1 = 19.1$ A

$I_2 = -9.02$ A

Ex.



$I = I_1 + I_2$

in parallel
having the same voltage

$\Delta V_\varepsilon + \Delta V_{R_1} = 0$

$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

$\varepsilon - I_1 R_1 = 0 \qquad \varepsilon - I_2 R_2 = 0$

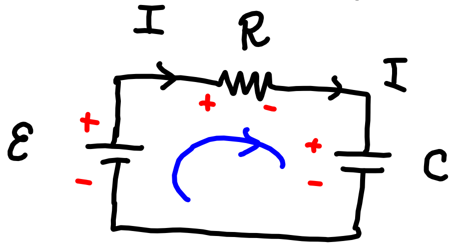
$\Rightarrow \quad I_1 = \frac{\varepsilon}{R_1} \qquad I_2 = \frac{\varepsilon}{R_2}$

$$I = \frac{\varepsilon}{R_1} + \frac{\varepsilon}{R_2} = \varepsilon\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \varepsilon \frac{1}{R_{eq}}$$

Circuits with capacitor + resistor:

$Q = CV$ $\quad$ $V = IR$

$V = \frac{q}{C}$

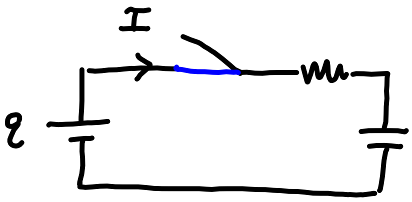


$\mathcal{E} = IR + \frac{q}{C}$

$\Delta V_{\mathcal{E}} + \Delta V_R + \Delta V_C = 0$

$+\mathcal{E} - IR - \frac{q}{C} = 0$

Ex.



at $t=0$ close the switch

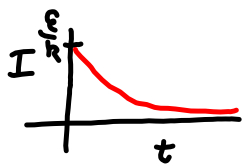
what is the current I and charge q?

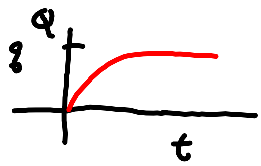
$q = 0 \quad I = \frac{\mathcal{E}}{R}$

after a long time $t \to \infty$

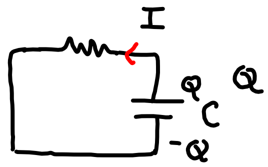
what is I, q?

$I = 0 \qquad q = C\mathcal{E} = Q$





Discharging the capacitor



$$\underset{\parallel}{I} R + \frac{q}{C} = 0 \qquad \text{in time } \Delta t$$

$$\frac{\Delta q}{\Delta t} R + \frac{q}{C} = 0 \Rightarrow \Delta q = -\frac{q}{RC}\Delta t$$

in time Δt : $q(\Delta t) = q_0 + \Delta q$ ← initial charge $= Q$

$$q(\Delta t) = q_0 - \frac{q_0 \Delta t}{RC} = q_0\left(1 - \frac{\Delta t}{RC}\right)$$

$$q(2\Delta t) = q(\Delta t)\left(1 - \frac{\Delta t}{RC}\right) = q_0\left(1 - \frac{\Delta t}{RC}\right)^2$$

$$q(3\Delta t) = q(2\Delta t)\left(1 - \frac{\Delta t}{RC}\right) = q_0\left(1 - \frac{\Delta t}{RC}\right)^3$$

$t = n\Delta t$

$$q(t) = q_0\left(1 - \frac{\Delta t}{RC}\right)^n = q_0\left(1 - \frac{t}{nRC}\right)^n$$

binomial expansion

$$\left(1 - \frac{t}{nRC}\right)^n = 1 - \frac{t}{nRC}n + \frac{n(n-1)}{2!}\frac{t^2}{n^2R C^2} + \dots$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots$$

$$= 1 - \frac{t}{RC} + \frac{1}{2!}\left(\frac{t}{RC}\right)^2 - \frac{1}{3!}\left(\frac{t}{RC}\right)^3 + \dots$$

$$= e^{-t/RC} \qquad e = 2.71\dots$$

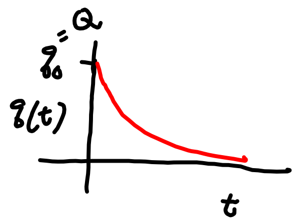
$$q(t) = q_0 e^{-t/RC}$$

$RC = t_0$ = characteristic time

$$= \frac{q_0}{e} \quad \text{if } t = t_0$$

$t = 2t_0$ $\quad q(t) = q_0/e^2$

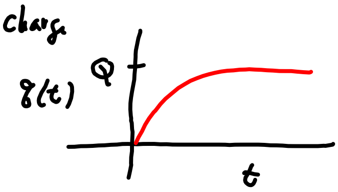
$t = 3t_0$ $\quad q(t) = q_0/e^3$



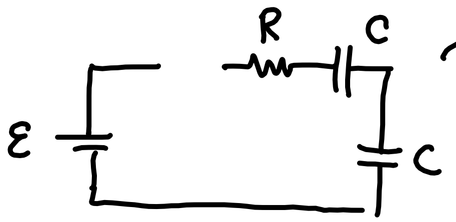
charging up the capacitor :

$$q(t) = Q\left(1 - e^{-t/RC}\right)$$

↑ final charge

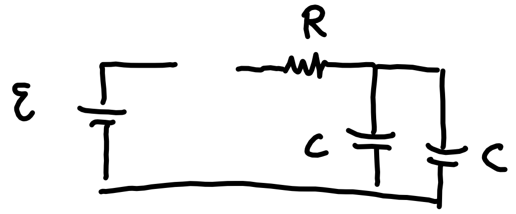


Ex. 1

A) 

$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C}$

$C_{eq} = \frac{C}{2}$

B) 

$C_{eq} = 2C$

Which configuration, when the switch is closed, charges up the capacitors the fastest?

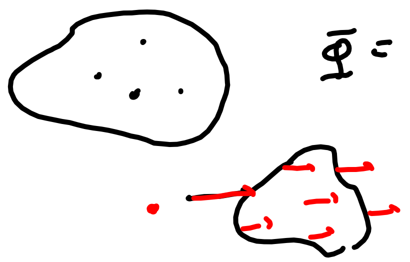
Review Chap 17–19

17) Coulomb force : for 3 charges

Electric field : $\vec{F} = q\vec{E}$

pt charge, charge plate, spherical shell

Gauss' Law : electric flux $\Phi = AE$



$\Phi = 4\pi k\, q_{enclose} = \epsilon_0\, q_{enclosed}$

$\Phi = 0$

18) Electric potential

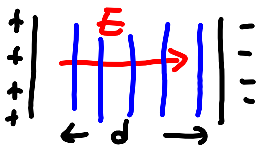
$$U = qV$$

potential energy (U) — electric potential (V)

produced by pt charge, charged plate.

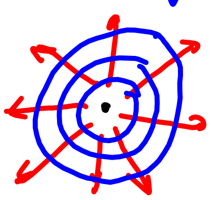
a) V decreases in the direction of E.

Equipotential surfaces



$\Delta V = Ed$

b)



$V = k\frac{q}{r}$

c) Capacitor $Q = CV$ $U = \frac{1}{2}QV$

energy in a capacitor

Simple capacitance circuit. $U = Vu$ $u = \frac{1}{2}\epsilon_0 E^2$

dielectric $E = E_0/K$ $C = KC_0$

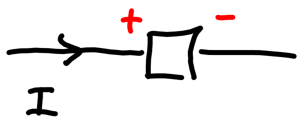
(E_0: vacuum)

19) Currents + circuits

$I = \frac{\Delta Q}{\Delta t}$ $V = IR$ $P = IV$

↑ voltage across the resistor ↖ power dissipated in a resistor

Apply Kirchhoff's Rule to solve any two-loop circuit



Sum of voltage difference = 0

RC time scales, charging + discharging a capacitor.