No class on Monday

Make-up class on Tuesday 7 pm

Project 2 due Thursday, 5 pm

in my mailbox

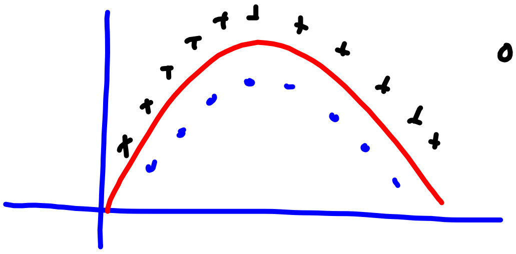
No hand-written response in all assignments

1) # 32

out the

exact ti:

Proj. 1



average

or

alternate.

Algorithms $\det M = 1$, $\det M \neq 1$

volume preserving, Runge-Kutta.

Symplectic,

1) Verlet algorithm: $\vec{r}_1 = \vec{r}_0 + \Delta t \vec{v}_0 + \frac{1}{2}\Delta t^2 \vec{a}_0$

$\vec{r}_{-1} = \vec{r}_0 - \Delta t \vec{v}_0 + \frac{1}{2}\Delta t^2 \vec{a}_0$

$\vec{r}_1 + \vec{r}_{-1} = 2\vec{r}_0 + \Delta t^2 \vec{a}_0$

$\det M = 1$ $\boxed{\vec{r}_{n+1} = 2\vec{r}_n - \vec{r}_{n-1} + \Delta t^2 \vec{a}_n}$ need to know two previous positions.

identical to Cromer, if the two initial pos. are the same.

multi-step algorithms

not self-starting

Cromer: $\vec{v}_{n+1} = \vec{v}_n + \Delta t \vec{a}_n$

$\vec{r}_{n+1} = \vec{r}_n + \Delta t \vec{v}_{n+1} \Rightarrow \vec{r}_n = \vec{r}_{n-1} + \Delta t \vec{v}_n$

$= \vec{r}_n + \Delta t (\vec{v}_n + \Delta t \vec{a}_n)$

$= \vec{r}_n + \Delta t \vec{v}_n + \Delta t^2 \vec{a}_n$

$= 2\vec{r}_n - \vec{r}_{n-1} + \Delta t^2 \vec{a}_n$

Velocity-form of the Verlet

dstM = 1

$$\vec{r}_{n+1} = \vec{r}_n + \Delta t\, \vec{v}_n + \frac{1}{2}\Delta t^2 \vec{a}_n \qquad \vec{a}_n = \vec{a}(\vec{r}_n)$$

$$\vec{v}_{n+1} = \vec{v}_n + \Delta t\, \frac{1}{2}\left(\vec{a}(\vec{r}_n) + \vec{a}(\vec{r}_{n+1})\right)$$

Second order in Δt

Leap-frog algorithm:

$$\vec{r}_1 = \vec{r}_0 + v_0 \Delta t + \frac{1}{2}\Delta t^2 \vec{a}_0 \qquad \vec{v}_1 = v_0 + \Delta t\, \vec{a}_0$$

$$\vec{r}_{-1} = \vec{r}_0 - v_0 \Delta t + \frac{1}{2}\Delta t^2 \vec{a}_0 \qquad \vec{v}_{-1} = v_0 - \Delta t\, \vec{a}_0$$

$$\vec{r}_1 - \vec{r}_{-1} = 2 v_0 \Delta t \qquad \vec{v}_1 - \vec{v}_{-1} = 2\Delta t\, \vec{a}_0$$

r

$-1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$

v

$$\vec{v}_2 = \vec{v}_0 + 2\Delta t\, \vec{a}_1$$

$$\vec{v}_{n+2} = \vec{v}_n + 2\Delta t\, \vec{a}(\vec{r}_{n+1})$$

the same as Cromer

structurally the same as:

$$\vec{v}_{n+1} = \vec{v}_n + \text{"}\Delta t\text{"}\, \vec{a}(\vec{r}_n)$$

How to develop algorithms with $\det M = 1$

↪ canonical transformation

Four formulations of classical mechanics

1) Newtonian: $\vec{F} = m\vec{a} \Rightarrow \frac{d^2\vec{r}}{dt^2} = \frac{\vec{F}(\vec{r})}{m}$

basis for Runge Kutta algorithm

2) Lagrangian: variational

↪ generalize coordinates (angles)

" momentum

3) Hamiltonian: momentum is a fundamental degree of freedom

ind. of $\vec{r}$.

$$\frac{d\vec{r}}{dt} = \frac{\vec{p}}{m} \; !$$

$$\underbrace{\frac{d\vec{p}}{dt} = -\frac{\partial H}{\partial \vec{r}} \qquad = \frac{\partial H}{\partial \vec{p}}}_{\text{Hamilton's Eq.}}$$

transformations of $(\vec{r}, \vec{p}) \to (\vec{r}', \vec{p}')$
which preserves the form of Han. Eq.
are canonical transformation: $\det M = 1$

4) Poissonian: instead of equations of motion,
operator form of the solution!