Your Name _____

Physics 606: Fall Semester 2018

Homework #9

Problem 1:

A small perturbation, $W = ax^4$, is applied to a harmonic oscillator with force constant k and reduced mass m. Compute the first-order correction to the eigenenergies and first nonvanishing correction to the wave functions

Problem 2:

Consider the Hamiltonian $H = H_0 + V(x, y)$, where $H_0 = (p_x^2 + p_y^2)/2m + mw^2(x^2 + y^2)/2$ is the 2D HO Hamiltonian, and where $V(x, y) = \lambda mw^2 xy$ is a perturbation.

(i) Find the exact ground state
(ii) Using the second-order perturbation, calculate the ground state energy.

Problem 3

Consider a hydrogen atom placed in a uniform static electric field ε that points along the $\hat{z}$-direction. The term that corresponds to this interaction in the Hamiltonian is

$$W = -e\varepsilon z$$

Note that for the electric fields typically produced in a laboratory, the condition $W \ll H_0$ is satisfied. The appearance of the perturbation removes the degeneracy from some of the hydrogen states. This phenomenon is called *Stark effect*. Calculate the Stark effect for $n = 2$ in a hydrogen atom.

Problem 4:(11.7 of Baym)

Estimate the ground state energy of the hydrogen atom using a three-dimensional harmonic oscillator ground state wave function as a trial function.

Problem 5:(11.10 of Baym)

Estimate the ground state energy of a particle in the one-dimensional well

$$V(x) = \begin{cases} \frac{1}{2}mw_0^2(x-a)^2, & for \;\; x > 0 \\ \\ \infty, & for \;\; x < 0 \end{cases}$$

Where $a > 0$. Is the energy greater of less than $\hbar w_0/2$.