Print Name: ________________________________

PHYSICS 606 : FALL SEMESTER 2018

Homework #8

1. Show that (9-8) is a Green's function satisfying (9-5)

2. a) If the potential is translationally invariant such that $v(\mathbf{r}) = v(\mathbf{r}+\mathbf{a})$, from (9-9) show that $\psi_{\mathbf{k}}(\mathbf{r})$ is a Block wave satisfying

$$\psi_{\mathbf{k}}(\mathbf{r}+\mathbf{a}) = \mathrm{e}^{i\mathbf{k}\cdot\mathbf{a}}\psi_{\mathbf{k}}(\mathbf{r})$$

b) In this case, use (9-12) to show that the scattering amplitude $f_{\mathbf{k}}$ vanishes unless $\mathbf{k}' = \mathbf{k}+\mathbf{q}$ where $\mathbf{q}$ is a reciprocal vector such that

$$\mathbf{q}\cdot\mathbf{a} = 2\pi n$$

for some integer n.

c) Show that b) remains true in the Born approximation.

3. a) For an infinite spherical wall of radius b, find $\tan(\delta_\ell)$ for all ℓ in the low energy limit of $kb \to 0$.

b) In the low energy limit as defined above, what is the total scattering cross-section? Why doesn't it agree with the simple geometric cross-section of πb^2?

4. A complicated potential is known to have scattering length a. Suppose we want to approximate this by use of a pseudopotential

$$V(\mathbf{r}) = C\delta(\mathbf{r}) = C\delta(x)\delta(y)\delta(z),$$

what must be the constant C so that when this pseudopotential is used in the Born approximation, we obtain the same scattering length? (C can be a function of a.)

5. The integral form of the Schrodinger equation for the partial wave radial solution can be shown to be given by

$$R_\ell(k,r) = j_\ell + \int_0^\infty G_\ell(k,r,r')U(r')R_\ell(k,r')r'^2 dr'$$

where $U(r) = 2mV(r)/\hbar^2$, $E = \hbar^2 k^2/(2m)$ and

$$G_\ell(k,r,r') = -ikj_\ell(kr_<)h_\ell(kr_>),$$

with the phase-shift determined from

$$\frac{1}{k}\mathrm{e}^{i\delta_\ell}\sin\delta_\ell = -\int_0^\infty j_\ell(kr)U(r)R_\ell(k,r)r^2 dr.$$

a) For a delta function, radial shell potential

$$V(r) = -\lambda\frac{\hbar^2}{2m}\delta(r-a),$$

show that the partial wave scattering amplitude is

$$\mathrm{e}^{i\delta_\ell}\sin(\delta_\ell) = \frac{\lambda aka|j_\ell(ka)|^2}{1 - i\lambda akaj_\ell(ka)h_\ell(ka)}$$

b) For $\ell = 0$, show that the binding energy is $-\hbar^2\kappa^2/2m$ with κ given by

$$2\kappa a = \lambda a(1 - \mathrm{e}^{-2\kappa a}).$$

(Compare this with your results on the two delta-functions potential in 1D).