Print Name: ______________________________

PHYSICS 606 : FALL SEMESTER 2018

Homework #4

1) Show that in the case of the potential well problem, with $V = -V_0$, in the limit of very small V_0, the bound state energy is given by (Baym, 4.65)

$$E = -\frac{1}{2}\frac{m}{\hbar^2}V_0^2 a^2.$$

2) For the finite potential barrier/well problem, we have derived the transfer matrix

$$\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} \cos\xi - i\frac{\lambda}{2}\sin\xi & i\frac{\eta}{2}\sin\xi \\ -i\frac{\eta}{2}\sin\xi & \cos\xi + i\frac{\lambda}{2}\sin\xi \end{pmatrix} \begin{pmatrix} F \\ G \end{pmatrix} \tag{1}$$

where

$$\lambda = r + \frac{1}{r} \qquad \eta = \frac{1}{r} - r \qquad r = \frac{k'}{k} \qquad \xi = k'a$$

and

$$\frac{\hbar^2 k^2}{2m} = E \qquad \frac{\hbar^2 k'^2}{2m} = E - V$$

Consider the limit of $V \to \infty$ and $a \to 0$ such that $aV = v$ is finite. This then corresponds to the delta function potential $v\delta(x)$.

a) By going to this limit, derive the transfer matrix for the delta function potential.

b) What is the transmission amplitude $S(E)$?

c) By setting $1/S(E) = 0$, find the bound state energy E for $v = -v_0$.

3) (Essential Baym 4.11) Use your delta function transfer matrix in **2)** to solve the case of a two delta-function potential

$$V(x) = v\delta(x) + v\delta(x-a).$$

a) Let the transfer matrix for a single delta-function be denoted simply as

$$\begin{pmatrix} 1+\gamma & \gamma \\ -\gamma & 1-\gamma \end{pmatrix} \tag{2}$$

what is the transfer matrix for the two delta-function case above?

b) Let $k = i\kappa$ and $v = -v_0$. Show that there are now two solutions to $1/S(E) = 0$ which determine κ and hence E. What are these two equations for determining κ? Indicate which equation is for the even state and which one is for the odd state. You need not solve for κ analytically.

c) For the even state, find the bound state energy analytically for $mv_0a/\hbar^2 < 1$. For the odd state, find the minimum value of v_0 for which a bound state is possible.

d) Find the bound state energy for both states when $a >> \hbar^2/mv_0$. Explain physically why the energies approach each other at large a.

4)(Baym, 4.14) Compute the expectation value of the following operators between states of m and n: x, p, x^2, p^2, px, xp, *i.e.* $\langle m|x^2|n\rangle$, etc.. Also compute $\langle 0|x^{10}|0\rangle$.