Name: ______________________________

PHYSICS 619 : SPRING SEMESTER 2019

Project #3: Time-Dependent Algorithms

1. Let's warm-up to the planar restricted three-body problem by first solving a 2D fixed two-center gravitational problem with equation of motion
$$\frac{d^2\mathbf{r}}{dt^2} = -\frac{1}{2}\frac{(\mathbf{r}-\mathbf{r}_1)}{d_1^3} - \frac{1}{2}\frac{(\mathbf{r}-\mathbf{r}_2)}{d_2^3},$$
where $d_1 = |\mathbf{r}-\mathbf{r}_1|$, $d_2 = |\mathbf{r}-\mathbf{r}_2|$ and $\mathbf{r}_1 = (-0.5, 0.0)$, $\mathbf{r}_2 = (0.5, 0.0)$. The situation can be regarded as two stars of equal mass $m = 1/2$ (in some units) located on the x-axis at $x = 0.5$ and $x = -0.5$. Starting the third body at the origin with $\mathbf{v}_0 = (v_0\cos\theta, v_0\sin\theta)$, determine a pair of v_0 and θ that will yield a closed figure 8 orbit encircling both stars. Use any second-order algorithm. Run the orbit 5 times to show that it is actually closed. Hand in this graph. (Optional, find any other closed orbit that intertwiningly encircles both stars.)
2. The restricted three-body problem. First, define a subroutine that gives the time-dependent acceleration:
$$\mathbf{a}(\mathbf{r}, t) = -\frac{1}{2}\frac{\mathbf{r}-\mathbf{r}_1(t)}{d_1^3} - \frac{1}{2}\frac{\mathbf{r}-\mathbf{r}_2(t)}{d_2^3},$$
where $d_1 = |\mathbf{r}-\mathbf{r}_1(t)|$, $d_2 = |\mathbf{r}-\mathbf{r}_2(t)|$ and now
$$\mathbf{r}_1(t) = (\,-0.5\cos(t), -0.5\sin(t)\,),$$
$$\mathbf{r}_2(t) = (\,0.5\cos(t), 0.5\sin(t)\,).$$
3. Implement the second-order PV (Position-Verlet) algorithm, also as a subroutine, corresponding to
$$\mathcal{T}^{(2)}(\Delta t, t) \equiv e^{\frac{1}{2}\Delta t\widetilde{T}} e^{\Delta t V} e^{\frac{1}{2}\Delta t\widetilde{T}}$$
by calling $\mathbf{a}(\mathbf{r}, t)$. Note that t in $\mathcal{T}^{(2)}(\Delta t, t)$ is always the initial time at the start of the algorithm. You must advance time and set $t = t + c_i\Delta t$ whenever you encounter $e^{c_i\Delta t D}$ in the algorithm, regardless whether you need to call $\mathbf{a}(\mathbf{r}, t)$. At the end of the algorithm, t is always advaned to $t + \Delta t$, for the start of the next iteration.
4. Implement the fourth order FR algorithm, also as a subroutine, corresponding to
$$\mathcal{T}_4(\Delta t, t) \equiv \mathcal{T}_2\Big(\frac{\Delta t}{2-s}, t_2\Big)\mathcal{T}_2\Big(\frac{-s\,\Delta t}{2-s}, t_1\Big)\mathcal{T}_2\Big(\frac{\Delta t}{2-s}, t\Big)$$
with $s = 2^{\,1/3}$, $t_1 = t + \Delta t/(2-s)$, and $t_2 = t_1 - s\,\Delta t/(2-s)$. At the end of this algorithm, one also has time advanced, $t \to t + \Delta t$. For this algorithm, you call the previously defined second order algorithm 3 times. Starting at the initial postion $\mathbf{r}_0 = (0.0, 0.058)$ with $\mathbf{v}_0 = (0.49, 0)$, plot the resulting figure. Run both algorithms at Δt=0.005 for slightly more than 9π in time and see how they compare. Hand in these two plots. Make sure that the orbit is closed and that the scale of the figure is not distorted.
5. Plot the trajectory in the co-rotating frame, by simply counter-rotate the trajectory when plotting: ($\mathbf{r} = (x, y)$)

```
XPLOT= COS(T)*X+SIN(T)*Y
YPLOT=-SIN(T)*X+COS(T)*Y.
```

`T` is final time after each run of the algorithm. Remember to call SIN and COS only once in each iteration, store and used them when needed. Do this only for the FR algorithm. Hand in this plot and one chaotic orbit. (This is easy, almost any starting point will do.) Try to find one that is *most* chaotic encircling both stars. Plot this only in the co-rotating frame.