Your Name ____________________

Phys. 606 **Homework 2** (due 9/12)

Problem 1 Given the Lagrangian

$$L = \frac{1}{2}m\mathbf{v}^2 + \frac{q}{c}\mathbf{v}\cdot\mathbf{A}(\mathbf{r}), \quad \text{where} \quad \mathbf{v} = \frac{d\mathbf{r}}{dt},$$

show that the Euler-Lagrange equation reproduces the Lorentz force law,

$$m\frac{d^2\mathbf{r}}{dt^2} = \frac{q}{c}\mathbf{v}\times\mathbf{B}(\mathbf{r}), \quad \text{where} \quad \mathbf{B}(\mathbf{r}) = \nabla\times\mathbf{A}(\mathbf{r}).$$

Problem 2 (3.8 of Baym)

Use

$$K(\mathbf{r}t, \mathbf{r}'t) = \left(\frac{m}{2\pi i\hbar(t-t')}\right)^{3/2} \exp\left[\frac{im(\mathbf{r}-\mathbf{r}')^2}{2\hbar(t-t')}\right]$$

and integrate explicitly to show that

$$\int d^3r_2 K(\mathbf{r}_1t_1, \mathbf{r}_2t_2)K(\mathbf{r}_2t_2, \mathbf{r}_3t_3) = K(\mathbf{r}_1t_1, \mathbf{r}_3t_3)$$

Problem 3 (3.9 of Baym)

(a) Show that the free particle propagator $K(\mathbf{r}t, \mathbf{r}'t')$ obeys the free particle Schrödinger equation:

$$\left(i\hbar\frac{\partial}{\partial t} + \frac{\hbar^2}{2m}\nabla^2\right)K(\mathbf{r}t, \mathbf{r}'t') = 0.$$

(b) Suppose that a particle is acted on by a potential $v(\mathbf{r}, t)$, and that its wave function at t_0 is $\psi_0(\mathbf{r})$, show that the wave function of the particle at the later time t is given as the solution to the integral equation

$$\psi(\mathbf{r}, t) = \int d^3r' K(\mathbf{r}t, \mathbf{r}'t_0)\psi_0(\mathbf{r}') + \frac{1}{i\hbar}\int_{t_0}^{t} dt' \int d^3r' K(\mathbf{r}t, \mathbf{r}'t')v(\mathbf{r}', t')\psi(\mathbf{r}', t')$$

Problem 4 (3.10 of Baym)

(a) The probability current density $\mathbf{j}(\mathbf{r}, t)$ is given in terms of the wave function by

$$\mathbf{j}(\mathbf{r}, t) = \frac{1}{2m}\left[\psi^*(\mathbf{r}, t)\frac{\hbar}{i}\nabla\psi(\mathbf{r}, t) - \frac{\hbar}{i}\nabla\psi^*(\mathbf{r}, t)\psi(\mathbf{r}, t)\right].$$

Show directly from the Schrödingers equation that the probability density,

$P(\mathbf{r},t) = |\psi(\mathbf{r},t)|^2$ and the probability current density obey the continuity equation

$$\frac{\partial}{\partial t}P(\mathbf{r},t) + \nabla \cdot \mathbf{j}(\mathbf{r},t) = 0.$$

(b) What is the form for $\mathbf{j}(\mathbf{r},t)$ when there is a magnetic field present specified by the vector potential $\mathbf{A}(\mathbf{r},t)$?

Problem 5 (3.11 of Baym) Consider a particle of charge e traveling in the eletromagnetic potentials

$$\mathbf{A}(\mathbf{r},t) = \nabla\lambda(\mathbf{r},t), \qquad \phi(\mathbf{r},t) = \frac{1}{c}\frac{\partial\lambda(\mathbf{r},t)}{\partial t}$$

where $\lambda(\mathbf{r},t)$ is an arbitrary scalar function.

(a) What are the electromagnetic fields described by these potentials ?

(b) Show that the wave function of the particle is given by

$$\psi(\mathbf{r},t) = exp\left[-\frac{ie}{\hbar c}\lambda(\mathbf{r},t)\right]\psi^{(0)}(\mathbf{r},t)$$

where $\psi^{(0)}(\mathbf{r},t)$ solves the Schrödinger equation with $\lambda = 0$.

(c) Let $v(\mathbf{r},t) = e\phi(t)$ be a spatially uniform time varying potential. Show that

$$\psi(\mathbf{r},t) = exp\left[-\frac{ie}{\hbar c}\int_{-\infty}^{t}\phi(t')dt'\right]\psi^{(0)}(\mathbf{r},t).$$

[why is the lower limit on the integral $-\infty$?]