Name: ________________________________

PHYSICS 619 : SPRING SEMESTER 2019

Project #2: Symplectic algorithms

1 a) Show analyticaly that both the Cromer and the Velocity-Verlet (VV) algorithms exactly conserve the angular momentum vector $\mathbf{L} = \mathbf{r} \times \mathbf{v}$. b) Show that the VV algorithm is area (phase space) preserving while the second-order Runge-Kutta algorithm is not.

2. Derive the other canonical second-order symplectic algorithm in an explicit algorithmic form (not velocity-Verlet). Are there any other second order symplectic algorithms with only one evaluation of the force?

3. For the same problem we solved in Project#1, compute the trajectory for 10 periods at $\Delta t = P/1000$ using the fourth-order Runge-Kutta-Nyström algorithm and

$$T_4 = (4/3)T_{VV}^2(\Delta t/2) - (1/3)T_{VV}(\Delta t),$$

where T_{VV} is the velocity-Verlet algorithm. Plot both trajectories in the same graph.

4. For the same problem we solved in Project#1 (see if you can only plot the small interval near the mid-period as in Project#1) plot $(E(t)/E_0 - 1)/(\Delta t)^4$ for 10 periods at $\Delta t = P/1000$ using the 4th order Runge-Kutta-Nyström algorithm and the 4th order Forest-Ruth algorithm where T_2 is the second-order algorithm of **2** (not velocity-Verlet). Plot both in one graph for comparison.

5. For $\Delta t = P/1000$, compare the rotation of the Laplace-Runge-Lenz vector

$$\mathbf{A} = \mathbf{p} \times \mathbf{L} - \hat{\mathbf{r}},$$

for the same two algorithms in **4** by computing

$$\frac{1}{(\Delta t)^4}\theta(t) = \frac{1}{(\Delta t)^4} \tan^{-1}\left[\frac{A_y(t)}{A_x(t)}\right]$$

over the same range as in 4. Also plot both in a single graph.