Print Name: ______________________________

PHYSICS 606 : FALL SEMESTER 2018

Homework #10

1. A 1D harmonic oscillator is in its ground state for $t < 0$. For $t \geq 0$, it is subjected to a time-dependent but uniform *force* in the x-direction

$$F(t) = F_0 \mathrm{e}^{-t/\tau}.$$

   (a) Using first order time-dependent perturbation theory, determine the probability of finding the particle in its first excited state.

   (b) What is the probability in the limit of $t \to \infty$ (τ finite)?

   (c) What is the probability of finding the particle in any other excited states?

2. Do Baym 12.2, and you may assume that the proton is fixed at the origin.

3. Derive the following key result for understanding Bell's inequality.

   Let

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 0 \\ 1 \end{pmatrix}_A \begin{pmatrix} 1 \\ 0 \end{pmatrix}_B - \begin{pmatrix} 1 \\ 0 \end{pmatrix}_A \begin{pmatrix} 0 \\ 1 \end{pmatrix}_B\right].$$

   Let

$$\hat{A} = \hat{\mathbf{a}} \cdot \boldsymbol{\sigma} \quad \hat{B} = \hat{\mathbf{b}} \cdot \boldsymbol{\sigma}$$

   be two spin operators pointing in the directions of unit vectors $\hat{\mathbf{a}}$, $\hat{\mathbf{b}}$ and act on particles A and B respectively.

   (a) Let $\hat{\mathbf{a}} = (\sin\theta_1 \cos\phi_1, \sin\theta_1 \sin\phi_1, \cos\theta_1)$ (similarly $\hat{\mathbf{a}} \to \hat{\mathbf{b}}$, $1 \to 2$), and

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

   what are the two 2×2 matrices $\hat{A}$ and $\hat{B}$? (in terms of $\mathrm{e}^{\pm i\phi}$)

   (b) Now derive the key result that (θ_{ab} is the angle between $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$)

$$\langle\psi|\hat{A}\hat{B}|\psi\rangle = -\hat{\mathbf{a}} \cdot \hat{\mathbf{b}} = -\cos\theta_{ab}$$

4. A simple hidden variable theory (HVT) for the spin variables is to define

$$A(\hat{\mathbf{a}}, \hat{\boldsymbol{\lambda}}) = \mathrm{sign}(\hat{\mathbf{a}} \cdot \hat{\boldsymbol{\lambda}}) \qquad B(\hat{\mathbf{b}}, \hat{\boldsymbol{\lambda}}) = -\mathrm{sign}(\hat{\mathbf{b}} \cdot \hat{\boldsymbol{\lambda}})$$

   where $\hat{\boldsymbol{\lambda}}$ is a uniform random unit vector. Take $\hat{\mathbf{a}} = (1, 0, 0)$, $\hat{\mathbf{b}} = (\cos\theta_{ab}, \sin\theta_{ab}, 0)$, $\hat{\boldsymbol{\lambda}} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$, and show that according to this HVT, the spin correlation function as defined below, is given by

$$\frac{1}{4\pi}\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta A(\hat{\mathbf{a}}, \hat{\boldsymbol{\lambda}}) B(\hat{\mathbf{b}}, \hat{\boldsymbol{\lambda}}) = -(1 - \frac{2\theta_{ab}}{\pi})$$