Print Name: ______________________________

PHYSICS 606 : **Two practice Final Exam questions on new materials**

Instructions
1. **Print** your name above.
2. **Ask questions** if you have the slightest doubt as to what you are asked to solve.
3. Indicate which part of the problem you are solving. **Show your work** and **circle your answer**.
4. You may continue your calculations or scratch work on the back of each sheet of paper

NOTE: Some formulas that you may need

$$[x_i, p_j] = i\hbar\delta_{ij} \quad a|n\rangle = \sqrt{n}|n-1\rangle \quad a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

$$x = \sqrt{\frac{\hbar}{2m\omega}}(a + a^\dagger) \quad p = -i\sqrt{\frac{\hbar m\omega}{2}}(a - a^\dagger) \qquad \nabla^2 = \frac{1}{r}\frac{\partial^2}{\partial r^2}r - \frac{\mathbf{L}^2/\hbar^2}{r^2}$$

$$\mathrm{e}^{\varepsilon A}B\mathrm{e}^{-\varepsilon A} = B + \varepsilon[A, B] + \frac{\varepsilon^2}{2!}[A, [A, B]] + \frac{\varepsilon^3}{3!}[A, [A, [A, B]]] + ...$$

$$\langle n'|x^2|n\rangle = \frac{\hbar}{m\omega}\Big(n + \frac{1}{2}\Big)\delta_{n',n} + \frac{\hbar}{2m\omega}\Big(\sqrt{n}\sqrt{n-1}\,\delta_{n',n-2} + \sqrt{n+2}\sqrt{n+1}\,\delta_{n',n+2}\Big)$$

$$\langle n'|x|n\rangle = \sqrt{\frac{\hbar}{2m\omega}}\Big(\sqrt{n}\,\delta_{n',n-1} + \sqrt{n+1}\,\delta_{n',n+1}\Big) \qquad E_N^{(1)} = \langle n|V|n\rangle \qquad E_N^{(2)} = \sum_{m\neq n}\frac{|\langle n|V|m\rangle|^2}{\epsilon_n - \epsilon_m}$$

$$\begin{aligned} f &= \frac{1}{k}\sum_{\ell=0}^{\infty}(2\ell+1)P_\ell(\cos\theta)\mathrm{e}^{i\delta_\ell}\sin\delta_\ell \\ &= -\frac{2m}{\hbar^2}\sum_{\ell=0}^{\infty}(2\ell+1)P_\ell(\cos\theta)\int_0^\infty r^2 dr v(r) j_\ell(kr) R_\ell(r) \end{aligned}$$

$$\frac{d\sigma}{d\Omega} = |f|^2 \qquad k\cot\delta_0 = -\frac{1}{a} \qquad j_0(x) = \frac{\sin(x)}{x}$$

$$P_{0\to n} = \Big|\frac{1}{i\hbar}\int_{t_0}^{t} dt' \mathrm{e}^{i(\epsilon_n - \epsilon_0)t'/\hbar}\langle n|V_{t'}|0\rangle\Big|^2$$

(DO NOT write anything below.)

1)______________________

2)______________________

3)______________________

4)______________________

5)______________________

6)______________________

Total

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5) Consider the 2D harmonic oscillator,

$$H_0 = \frac{1}{2m}(p_x^2 + p_y^2) + \frac{1}{2}m\omega^2(x^2 + y^2)$$

with eigenstates $|n_x, n_y\rangle = |n_x\rangle|n_y\rangle$ and energy

$$E = (n_x + n_y + 1)\hbar\omega.$$

The first excited state is doubly degenerate with states $|1, 0\rangle$ and $|0, 1\rangle$ having the same energy $E_1 = 2\hbar\omega$. If now H_0 is perturbed by an additional potential

$$V = m\omega^2\lambda xy,$$

a) what are the diagonal and off-diagonal matrix elements $\langle 1, 0|V|1, 0\rangle$, $\langle 0, 1|V|0, 1\rangle$, $\langle 1, 0|V|0, 1\rangle$, and $\langle 0, 1|V|1, 0\rangle$?
b) What are the eigenvalues of the above V-matrix?
c) What are the two new, non-degenerate eigenvalues splitted from E_1?
d) Show that if $H_0 + V$ is solved exactly and expanded to first order in λ, one recovers the two values in **c)**.

6) A 1D harmonic oscillator is in its first excited state for $t < 0$. For $t \geq 0$, it is subjected to a time-dependent potential of the form

$$V = (Ax + Bx^2)\mathrm{e}^{-\alpha t}$$

Use first order time-dependent perturbation theory and determine in the limit of $t \to \infty$:
a) The probability of finding the particle in the ground state.
b) The probability of finding the particle in the first excited state.
c) The probability of finding the particle in its second excited state.
d) The probability of finding the particle in its third excited state.