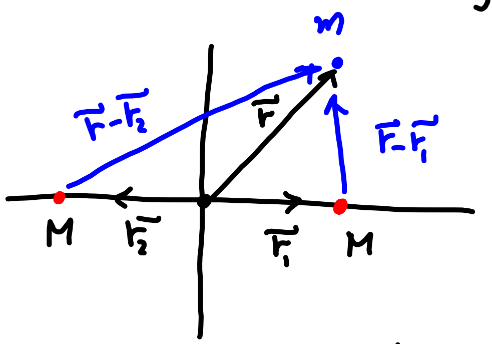
Solving for explicitly time-dependent force problems : $\vec{a}(\vec{r}, t)$

Gravitational 3-body problem $\leftrightarrow$ no analytical solution

Sun - earth - moon

$\hookrightarrow$ most orbits are chaotic !

2) Two-center problem - two gravitational attraction centers.



$$\cancel{m}\ddot{\vec{r}} = -GM\cancel{m}\frac{\vec{r}-\vec{r}_1}{|\vec{r}-\vec{r}_1|^3} - GM\cancel{m}\frac{\vec{r}-\vec{r}_2}{|\vec{r}-\vec{r}_2|^3}$$

choose :

$|\vec{r}_1| = |\vec{r}_2| = \frac{1}{2}$

$\vec{r}_1 = (\frac{1}{2}, 0)$

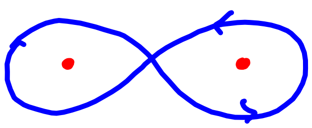
$\vec{r}_2 = (-\frac{1}{2}, 0)$

$$\boxed{\ddot{\vec{r}} = -\frac{1}{2}\left(\frac{\vec{d}_1}{|\vec{d}_1|^3} + \frac{\vec{d}_2}{|\vec{d}_2|^3}\right)}$$

where $\vec{d}_1 = \vec{r}-\vec{r}_1$, $\vec{d}_2 = \vec{r}-\vec{r}_2$

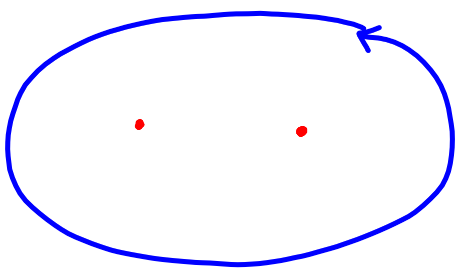
Possible orbits 1)

3)

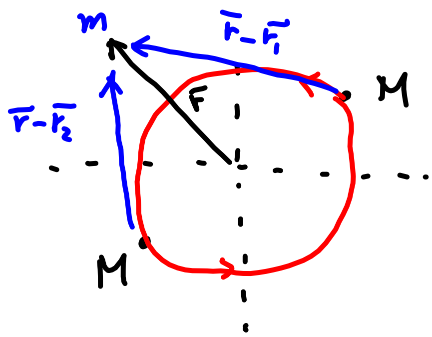


new Figure 8-orbit

2)



The circular-restricted 3-body problem



$m << M$

Assume that m has no affect on the motions of Ms.

$\vec{r}_1(t) = \frac{1}{2}(\cos(\omega t), \sin(\omega t))$

$\vec{r}_2(t) = -\vec{r}_1(t)$

$\omega = 1$

$$\ddot{\vec{r}} = \vec{a}(\vec{r}, t)$$

$$= -\frac{1}{2}\left(\frac{\vec{d}_1(t)}{|d_1(t)|^3} + \frac{\vec{d}_2(t)}{|d_2(t)|^3}\right)$$

where $d_1(t) = \vec{r} - \vec{r}_1(t)$, $d_2 = \vec{r} - \vec{r}_2(t)$

$\Rightarrow$ motion of m in a time-varying gravitational field.

Symplectic algorithms for solving
a time-dependent $\vec{a}(\vec{r}, t)$

For the case Sym 1a + 1b algorithm

1a)

Cromer

$t = 0$

1) $\vec{v}_1 = \vec{v}_0 + \vec{a}(\vec{r}_0, t)\Delta t$ ← evaluate $\vec{a}$ at the initial time

2) $\vec{r}_1 = \vec{r}_0 + \vec{v}_1 \Delta t$

3) $t = t + \Delta t$

$\Big\}$ for-loop

then repeat

1b)

$t = 0$

1) $\vec{r}_1 = \vec{r}_0 + \vec{v}_0 \Delta t$

2) $t = t + \Delta t$

3) $\vec{v}_1 = \vec{v}_0 + \vec{a}(\vec{r}_1, t)$ ← evaluate at the end-time.

$\Big\}$ for-loop

Second-order 2b (don't use 2a)

↑ position-Verlet ↑ velocity-Verlet

2b)

$$T_{2b} = e^{\frac{1}{2}\Delta t T} e^{\Delta t V} e^{\frac{1}{2}\Delta t T}$$

$t=0$

$$\vec{r}_1 = \vec{r}_0 + \frac{1}{2}\Delta t \vec{v}_0$$

$t = t + \frac{1}{2}\Delta t$

$$\vec{v}_1 = \vec{v}_0 + \Delta t \, \vec{a}(\vec{r}_1, t)$$

$$\vec{r}_2 = \vec{r}_1 + \frac{1}{2}\Delta t \, \vec{v}_1$$

$t = t + \frac{1}{2}\Delta t$

for-loop

update t after every updating of $\vec{r}$, with the coefficient of v.