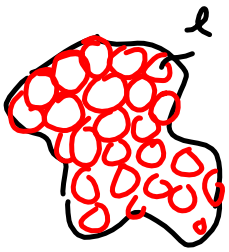
The end states of chaotic maps are self-similar fractal !

Fractal (non-integral) dimensions :

Given an object, to find its dimension,
cover it by a d-dimension "sphere" of radius $\ell$




$N(\ell)$ = # of ↗ that can cover the object.

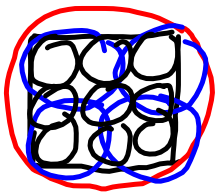
$\lim_{\ell \to 0} N(\ell) = \text{const} \left(\frac{1}{\ell}\right)^D$ ← dimension

$\ln N = D \ln\left(\frac{1}{\ell}\right)$

$$D = \lim_{\ell \to 0} \frac{\ln N}{\ln\left(\frac{1}{\ell}\right)}$$

Ex 1: unit sq.

$r = \sqrt{2}$



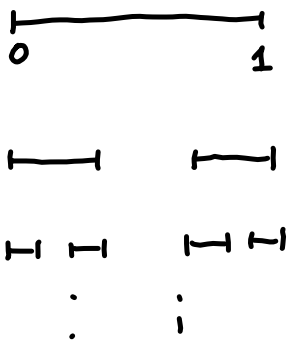
| $\ell$ | $N$ |
|---|---|
| 1 | 1 |
| $1/2$ | 4 |
| $1/3$ | 9 |
| ⋮ | ⋮ |
| $\frac{1}{n}$ | $n^2$ |

$\ell \to 0$
$n \to \infty$

$$D = \lim_{\ell \to 0} \frac{\ln(n^2)}{\ln(n)} = \frac{2\ln n}{\ln n} = 2$$

## Ex. 2 Cantor Set (throw out the middle 1/3)

$[0,1]$




| $\ell$ | $N$ |
|---|---|
| $1$ | $1$ |
| $1/3$ | $2$ |
| $1/9$ | $4$ |
| $\vdots$ | $\vdots$ |
| $1/3^n$ | $2^n$ |

$$\lim_{n\to\infty} D = \frac{\ln 2^n}{\ln 3^n} = \frac{n \ln 2}{n \ln 3} = 0.631\ldots$$