EPR (Einstein-Podolsky-Rosen) -(Bohm)

$A = e^-$
$B = e^+$

Positronium
$L=0, S=0$

The spins are correlated due to entanglement of the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(\begin{pmatrix}0\\1\end{pmatrix}_A \begin{pmatrix}1\\0\end{pmatrix}_B - \begin{pmatrix}1\\0\end{pmatrix}_A \begin{pmatrix}0\\1\end{pmatrix}_B \right) \qquad \vec{S}=0$$

$A = -1$ spin down $\qquad B = +1$ up
$\;\;\; = +1$ up $\qquad\qquad\;\; = -1$ down

Problems:

1) Realism: QM: the spin of A (or B) is NOT defined, until measurement.
CM: A must be either up or down while in flight.

2) locality: QM: spin correlation is perfect regardless of separation (faster than light.)
CM: action can only be local.

Identical twin ↔ perfect correlation
↪ same genes
↑
Hidden variable theory

randomly choose a spin ↑, ↓ then carry them apart.

John Bell : simple HVT, explains the spin correlation.

$$\rightarrow A(\hat{a},\hat{\lambda}) \qquad B(\hat{b},\hat{\lambda})$$

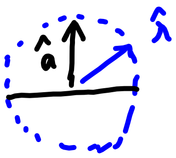
hidden variable corresponding to spin

$\neq \hat{b}$, same $\hat{\lambda}$

$$A(\hat{a},\hat{\lambda}) = \mathrm{sign}(\underset{\overset{\cos\theta}{\shortparallel}}{\hat{a}\cdot\hat{\lambda}}) = \pm 1$$

$$\vec{B}(\hat{b},\hat{\lambda}) = -\mathrm{sign}(\hat{b}\cdot\hat{\lambda}) = \pm 1$$

where $\hat{\lambda}$ is a uniform random unit vector



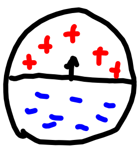
If $\hat{a}=\hat{b}$ ← EPR, explain the spin-correlation. (opposit)

what happen when $\hat{b} \neq \hat{a}$

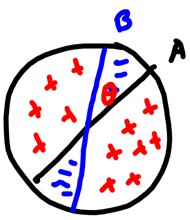
QM : $E(\hat{a},\hat{b}) = \langle\psi|\, \hat{a}\cdot\vec{\sigma}_A\ \hat{b}\cdot\vec{\sigma}_B\, |\psi\rangle = -\cos\theta_{ab}$

HVT : $E(\hat{a},\hat{b}) = \frac{1}{4\pi}\int d\Omega_\lambda\, A(\hat{a},\hat{\lambda})\, B(\hat{b},\hat{\lambda})$

difference



tilted



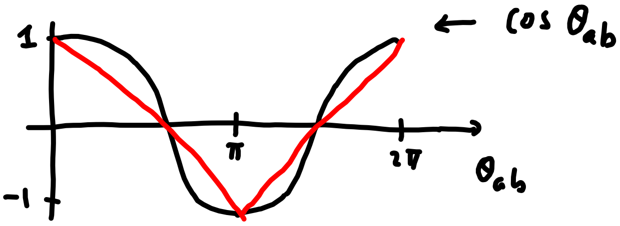
$$= -\left(1-\frac{2\theta_{ab}}{\pi}\right)$$

fraction of the spherical area $\frac{\theta}{2\pi}$

$$f(\theta) = -\frac{2\theta}{2\pi} + \frac{2\pi-2\theta}{2\pi}$$

$$= 1 - \frac{4\theta}{2\pi} = \left(1-\frac{2\theta}{\pi}\right)$$

Compare simple HVT vs QM



$f(\theta) = 1 - \frac{2\theta}{\pi}$

$-\pi < \theta < \pi$

$= -3 + \frac{2\theta}{\pi}$

$\pi < \theta < 2\pi$

form

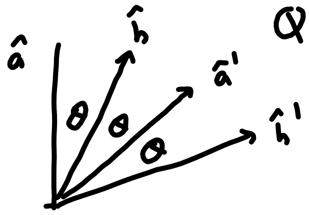
Bell's inequality : CHSH (clauser, Horne, shimony, Hot)

$A = A(\hat{a}, \lambda) \qquad B = B(\hat{b}, \lambda)$

$A' = A(\hat{a}', \lambda) \qquad B' = B(\hat{b}', \lambda)$

each can only be ± 1

$$-2 < \underbrace{A(B - B') + A'(B + B')}_{= S} < 2$$



QM : $S = \cos\theta_{ab} - \cos_{ab'} + \cos_{a'b} + \cos_{a'b'}$

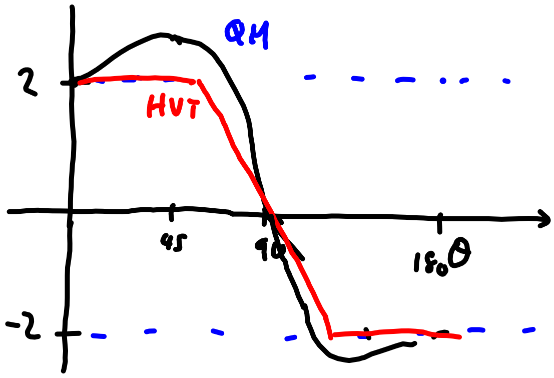
$= 3\cos\theta - \cos 3\theta$

HVT

$3f(\theta) - f(3\theta)$

$3\left(1 - \frac{2\theta}{\pi}\right) - \left(1 - \frac{6\theta}{\pi}\right)$

$= 2$



$\frac{6\theta}{\pi} = 2$

$\theta = \frac{2\pi}{6} = \frac{180}{3} = 60$

The max :

$-3\sin\theta + 3\sin 3\theta = 0$

$\sin\theta = \sin 3\theta$

at $\theta = 45°$

$S = 3\cos 45° - \cos(135°)$

$= 3\frac{1}{\sqrt{2}} - \left(-\frac{1}{\sqrt{2}}\right)$

$= \frac{4}{\sqrt{2}} = 2\sqrt{2}$

$= 2.828$

Bell-Kochen-Specker Thm: (Mermin)

no inequality is necessary to show that QM is nonlocal & non-"real"

rows & columns commute: simultaneous eigenvalues

$$\begin{array}{ccc} \sigma_x^A & \sigma_x^B & \sigma_x^A \sigma_x^B \\ \sigma_y^B & \sigma_y^A & \sigma_y^A \sigma_y^B \\ \sigma_x^A \sigma_y^B & \sigma_x^B \sigma_y^A & \sigma_z^A \sigma_z^B \end{array}$$

↗ row product $\underbrace{\sigma_x^A \sigma_y^A}_{i\sigma_z^A} \sigma_z^A \; \sigma_y^B \underbrace{\sigma_x^B}\sigma_z^B$

$i(\sigma_z^A)^2 \; (-i)(\sigma_z^B)^2 = 1$

column product $= -1$

by row : classical variable $= 1$

column " " $= -1$