Print Name: ______________________________

PHYSICS 606 : Sample Midterm Exam

**Exam I**

**Instructions**

1. **Print** your name above.
2. **Ask questions** if anything is unclear.
3. Indicated clearly which part of the problem you are doing. **Show your work** and **circle your answers**.
4. You may continue your calculations or scratch work on the back of each sheet of paper

**NOTE: Some formuli you may need**

$$[x_i, p_j] = i\hbar\delta_{ij} \quad a|n\rangle = \sqrt{n}|n-1\rangle \quad a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

$$x = \sqrt{\frac{\hbar}{2m\omega}}(a + a^\dagger) \quad p = -i\sqrt{\frac{\hbar m\omega}{2}}(a - a^\dagger)$$

$$\mathrm{e}^{\varepsilon A} B \mathrm{e}^{-\varepsilon A} = B + \varepsilon[A, B] + \frac{\varepsilon^2}{2!}[A, [A, B]] + \frac{\varepsilon^3}{3!}[A, [A, [A, B]]] + ...$$

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(Do not write anything below, for grading only.)

1)______________________

2)______________________

3)______________________

4)______________________

5)______________________

**Total**

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**1)** From the fundamental commutator relation $[x, p] = i\hbar$ and the product rule $[A, BC] = [A, B]C + B[A, C]$ compute

**a)** $[x, p^2]$

**b)** $[x, p^3]$

What then is

**c)** $[x, f(p)]$

if $f(p)$ has a power series expansion in $p$?

**2a)** Is the operator $\mathbf{r} \cdot (\mathbf{A} \times \mathbf{p})$ hermitian? $\mathbf{A}$ is just a numerical vector. (Note: $(\mathbf{A} \times \mathbf{p})_i = \epsilon_{ijk} A_j p_k$). If not, how can you modify it so that it is hermitian?

**2b)** Is the operator $\mathbf{p} \times (\mathbf{r} \times \mathbf{p})$ hermitian? If not, how can you modify it so that it is hermitian?

**3)** For the harmonic oscillator,

**a)** Compute $\langle 0|x^2|0\rangle$, $\langle 0|p^2|0\rangle$ and then $\sqrt{\langle 0|x^2|0\rangle}\sqrt{\langle 0|p^2|0\rangle}$.

**b)** What is $\sqrt{\langle n|x^2|n\rangle}\sqrt{\langle n|p^2|n\rangle}$ ?

**4)** Consider the Hamiltonian operator describing free-fall motion in one dimension,

$$H = \frac{1}{2m}p^2 + mgx.$$

**a)** Solve $x(t)$ as an Heisenberg operator directly by evaluating the operator expression

$$x(t) = \mathrm{e}^{iHt/\hbar}\, x(0)\, \mathrm{e}^{-iHt/\hbar}.$$

**b)** What is the variance of the position operator

$$\langle x^2(t)\rangle - \langle x(t)\rangle^2$$

as a function of time as compared to its initial variance?

**5.** The eigencondition for the ground state of a finite potential well of width $a$ and depth $V = -V_0$ is

$$\kappa = k' \tan(\frac{1}{2}k'a)$$

where

$$\hbar\kappa = \sqrt{2m(-E)}$$

$$\hbar k' = \sqrt{2m(E + V_0)}.$$

From this, deduce the ground state energy of an attractive delta-function well, where the strength of the delta-function potential $g$ is defined as

$$g = \lim_{a\to 0, V_0\to\infty} aV_0$$