Print Name: ________________________________

PHYSICS 606 : **Old Exam II**

READ and FOLLOW INSTRUCTION!

1. **Print** your name above.
2. **Ask questions** if anything is unclear.
3. Indicated which part of the problem you are doing. **Show your work** and **circle your answer**.
4. You may continue your calculations or scratch work on the back of each sheet of paper

NOTE: Some formulas you may need

$$[x_i, p_j] = i\hbar\delta_{ij} \quad a|n\rangle = \sqrt{n}|n-1\rangle \quad a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

$$A(t) = \mathrm{e}^{iHt/\hbar} A \mathrm{e}^{-iHt/\hbar} \qquad a|n\rangle = \sqrt{n}|n-1\rangle \qquad a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

$$a = \sqrt{\frac{m\omega}{2\hbar}}\Big(x + i\frac{p}{m\omega}\Big) \qquad a^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\Big(x - i\frac{p}{m\omega}\Big) \qquad \nabla^2 = \frac{1}{r}\frac{\partial^2}{\partial r^2}r - \frac{\mathbf{L}^2/\hbar^2}{r^2}$$

$$\begin{aligned} f &= \frac{1}{k}\sum_{\ell=0}^{\infty}(2\ell+1)P_\ell(\cos\theta)\mathrm{e}^{i\delta_\ell}\sin\delta_\ell \\ &= -\frac{2m}{\hbar^2}\sum_{\ell=0}^{\infty}(2\ell+1)P_\ell(\cos\theta)\int_0^\infty r^2 dr v(r) j_\ell(kr) R_\ell(r) \end{aligned}$$

$$\frac{d\sigma}{d\Omega} = |f|^2 \qquad k\cot\delta_0 = -\frac{1}{a} \qquad j_0(x) = \frac{\sin(x)}{x}$$

(DO NOT write anything below, for grading only.)

1)________________________

2)________________________

3)________________________

4)________________________

5)________________________

Total

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1. The angular momentum operators satisfy

$$[L_i, L_j] = i\hbar\epsilon_{ijk}L_k.$$

a) Show that

$$\langle \ell m|L_x|\ell m\rangle = 0 \quad \text{and} \quad \langle \ell m|L_y|\ell m\rangle = 0,$$

where $|\ell m\rangle$ are eigenstates of $\mathbf{L}^2$ and L_z.

b) What is $\langle \ell m|L_x^2 + L_y^2|\ell m\rangle$?

2. For the 1D harmonic oscillator

$$H = \frac{1}{2m} p_x^2 + \frac{1}{2} m\omega^2 x^2,$$

the ground state wave function is given by

$$\psi_0(x) = \Big(\frac{m\omega}{\pi\hbar}\Big)^{1/4} \exp\Big(-\frac{m\omega}{2\hbar} x^2\Big).$$

a) What is the ground state wave function of the 2D harmonic oscillator

$$H = \frac{1}{2m}(p_x^2 + p_y^2) + \frac{1}{2} m\omega^2 r^2$$

in terms of r where $r = \sqrt{x^2 + y^2}$?

b) What is the probability of finding the ground state particle outside of the classically allowed region in this case ?

3. The reduced radial Schrödinger equation for the hydrogen atom is

$$\frac{\hbar^2}{2m}\Big(-\frac{d^2}{dr^2}+\frac{\ell(\ell+1)}{r^2}\Big)u(r)-\frac{e^2}{r}u(r)=Eu(r).$$

a) For what values of r_0 and n is the following wave function an exact energy eigenstate?

$$u(r)=r^n \mathrm{e}^{-r/r_0}$$

Express r_0 in terms of the Bohr radius $a_0=\hbar^2/(me^2)$.

b) What is the resulting energy in units of Hartree H$= e^2/a_0$?

4. Consider low-energy s-wave scattering with energy $E = \hbar^2 k^2/(2m) \to 0$ from a spherical well with radius b and well-depth V_0 (*i.e.*, $V = -V_0$).

a) In the limit of $k \to 0$, $\delta_0 \to -ka$, where a is the s-wave scattering length, find a in the Born approximation.

b) In the limit of $b \to 0$ and $V_0 \to \infty$ such that $\frac{4\pi}{3} b^3 V_0 = C$, how is a related to C?