From measuring and classifying the stars to understanding their physics

• What we can measure directly:
  – Surface temperature and color
  – Spectrum
  – Apparent magnitude or intensity
  – Diameter of a few nearby stars
  – Distance to nearby stars

• What we need in order to understand them:
  – Mass
  – Luminosity (energy radiated per second)
We solved a 2-body problem and derived Kepler’s laws.

We applied them to the Solar system. How about stars?

\[
\mathbf{R} = \frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2}{m_1 + m_2} = 0 \quad \Rightarrow \quad \frac{m_1}{m_2} = \frac{r_2}{r_1} = \frac{a_2}{a_1}
\]
Binary stars!

The only direct way to measure stellar masses

Key to all properties of the stars: luminosity, age, size, lifetime
Term *binary* was first used by Sir Williams Herschel in 1802.

"If, on the contrary, two stars should really be situated very near each other, and at the same time so far insulated as not to be materially affected by the attractions of neighbouring stars, they will then compose a separate system, and remain united by the bond of their own mutual gravitation towards each other. This should be called a real double star; and any two stars that are thus mutually connected, form the binary sidereal system which we are now to consider."
More than 50% of all stars in our Milky Way are not single stars, but belong to binaries:

Pairs or multiple systems of stars which orbit their common center of mass.

If we can measure and understand their orbital motion, we can estimate the stellar masses.
The Center of Mass

Center of mass = balance point of the system.

Both masses equal => center of mass is in the middle, \( r_A = r_B \).

The more unequal the masses are, the more it shifts toward the more massive star.
Center of Mass
Animations of binary Stars:


http://abyss.uoregon.edu/~js/applets/eclipse/eclipse.htm
Estimating Masses

Arbitrary units:

\[ M_A + M_B = \frac{4\pi^2 a^3}{GP^2} \]

Only the total mass can be measured

Both semimajor axis and period of rotation must be known (very rare case)
Estimating Stellar Masses

Recall Kepler’s 3rd Law:

\[ P_y^2 = a_{AU}^3 \]

Valid for the Earth+Sun system: \( M_A + M_B = 1 \) solar mass, \( a = 1 \) AU, \( P = 1 \) year

Therefore, if we measure \( M \) in solar masses, \( a \) in AU and \( P \) in years, we can write

\[
M_A + M_B = \frac{a_{AU}^3}{P_y^2}
\]

\( (M_A \text{ and } M_B \text{ in units of solar masses}) \)
Example

Binary system with period of $P = 32$ years and separation of $a = 16$ AU:

$$M_A + M_B = \frac{16^3}{32^2} = 4 \text{ solar masses.}$$

How to measure period and separation?
Visual Binaries

The ideal case:

Both stars can be seen directly, and their separation and relative motion can be followed directly.
If semimajor axes of two elliptical orbits are visible, one can find the ratio of masses:

\[
\frac{m_1}{m_2} = \frac{a_2 \cos i}{a_1 \cos i} = \frac{a_2}{a_1}
\]

From 3rd Kepler’s law:

\[
m_1 + m_2 = \frac{4\pi^2}{GP^2} \left( \frac{a}{\cos i} \right)^3
\]
Visual binaries

The Castor system

The Sirius system

The two stars are separately visible in the telescope
Astrometric binary:

Only one star is seen; the companion is too faint, but its presence is detected by its gravitational influence on the primary star (oscillatory motion).

Wobbling motion of Sirius A
Binary Stars

Sirius A
brightest star in the sky
$m = -1.46$.

In 1844, Fraiedrich Bessel deduced it was a binary.

In 1862 Alvan Graham Clark discovered the companion.

Sirius B
$m = 8.30$
Spectroscopic Binaries

Usually, binary separation \( a \) can not be measured directly because the stars are too close to each other.

Stars are seen as a single point

However:

1) their SPECTRA are different, like different fingerprints;

2) Their spectral lines shift periodically because of Doppler effect. This allows us to measure their orbital velocities

3) Sometimes only one set of spectral lines is visible
The Castor system: a sextuple system!

The two stars are separately visible in the telescope; Each of them is a spectroscopic binary; In addition, there is a faint companion which is an eclipsing binary
The Doppler Effect

The light of a moving source is blue/red shifted by

\[ \frac{\Delta \lambda}{\lambda_0} = \frac{v_r}{c} \]

- \( \lambda_0 \) = actual wavelength emitted by the source
- \( \Delta \lambda \) = Wavelength change due to Doppler effect
- \( v_r \) = radial velocity (along the line of sight)

Blue Shift (to higher frequencies) | Red Shift (to lower frequencies)
The Doppler effect: apparent change in the wavelength of radiation caused by the relative motion of the source and detector

\[
\text{Shift } z = \frac{\text{Observed wavelength} - \text{Rest wavelength}}{\text{Rest wavelength}}
\]

\[
z = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{\Delta \lambda}{\lambda_0}
\]

Doppler effect:

\[
z = \frac{\Delta \lambda}{\lambda_0} = \frac{V_{rad}}{c}; \quad V_{rad} \ll c
\]

\[
\frac{V_{\text{radial}}}{c} = \frac{\Delta \lambda}{\lambda_0} = -\frac{\Delta \nu}{\nu_0}; \quad \frac{V_{\text{radial}}}{c} \ll 1
\]
The Doppler effect allows us to measure the source’s radial velocity.

\[ \frac{\Delta \lambda}{\lambda_0} = \frac{v_r}{c} \]
Spectroscopic Binaries

The approaching star produces blue shifted lines; the receding star produces red shifted lines in the spectrum.

Doppler shift → Measurement of radial velocities

→ Estimate of separation $a$

→ Estimate of masses
Spectroscopic binaries

Stars are seen as a single point

- Spectra of both stars are distinguishable
- Sometimes spectrum of only one star is seen
Spectroscopic Binaries (3)

Typical sequence of spectra from a spectroscopic binary system
Determining the orbital period
• Measure the orbital period
• Measure the radial component of the orbital velocities
• Can estimate the orbit size
• Can determine masses!

$$M_1 + M_2 = \frac{a^3}{P^2};$$

Where mass is in solar masses, $a$ in AU, $P$ in years
1. Below is a radial velocity curve for a spectroscopic binary. Estimate the mass of each star if the mass of the binary system is 6 solar masses.

\[
M_A \ d_A = M_B \ d_B
\]

\[V \sim 2\pi d/P\]  \quad \text{Assumes circular orbits!}

\[
\frac{M_A}{M_B} = \frac{d_B}{d_A} = \frac{V_B}{V_A} = \frac{1}{2}
\]

\[M_A + M_B = 6\]
Only the function of masses and inclination angle can be measured

Derivation of the mass function – chapter 7
THE PLANET CANNOT BE SEEN

...BUT

MOTIONS
OF THE STAR
BETRAY
ITS PRESENCE!
EARTH

- 450 km
- 9 cm/s

JUPITER

- 750 000 km
- 13 m/s

- 150 000 000 km
- 30 km/s

- 780 000 000 km
- 13 km/s
MOTIONS OF THE SUN VIEWED FROM A STAR 30 LIGHT YEARS AWAY

0.002” IS THE ANGULAR SIZE OF A MAN ON THE MOON OR A STANDARD NEWSPAPER FONT 300 KM AWAY

Unobservable!
STELLAR WOBBLE

RECEDING: REDDER

APPROACHING: BLUER

Doppler Shift due to Stellar Wobble
\[ \sigma = 11 \text{ ms}^{-1} \]
Hundreds of planets discovered
EXPECTED:

NEARLY CIRCULAR ORBITS
BIG PLANETS FAR AWAY FROM THE STAR
NO PLANETS BIGGER THAN JUPITER

DISCOVERED:

STRONGLY ELONGATED ORBITS
BIG PLANETS VERY CLOSE TO THE STAR
MANY PLANETS BIGGER THAN JUPITER
Planetary system of \( \nu \) And

- **B**: 0.06 AU, 4.5 days, 0.75 M\(_J\)
- **C**: 0.85 AU, 242 days, 2 M\(_J\)
- **D**: 2.5 AU, 3.5 years, 4 M\(_J\)
- **0.39 AU, 89 days**
- **0.73 AU, 228 days**
- **1 AU, 1 year**
- **1.54 AU, 1.9 years**

Source: Harvard-Smithsonian CfA
Habitable zones

- **Rho CrB**: 0.72 to 1.42 AU
- **47 UMa**: 0.76 to 1.90 AU
- **Gliese 876**: 0.112 to 0.221 AU
- **Ups And**: 1.25 to 2.49 AU

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Signs of life in the spectrum:
They are everywhere!

- 55 Cancri
- HD 16782
- 70 Virginis
- 47 Ursa Majoris
- E Boötis
- U Andromedae
- 3 Corona Borealis
- 16 Cygni B
- 31 Pegasi
- Gliese 576
- 14 Hercules
The Puzzle of Algol
John Goodricke 1764-1786

Explained Algol puzzle in 1783
Eclipsing binaries
Eclipsing Binaries

Usually, inclination angle of binary systems is unknown → uncertainty in mass estimates.

Special case:

Eclipsing Binaries

Here, we know that we are looking at the system edge-on!
The light curve of Algol
Measuring orbital period and diameters
Measuring diameters

\[ D = V_{\text{orb}} (t_2 - t_1) \]
Spectral Classification of Stars
Mass-Luminosity relation

- Detached main-sequence systems, B6 to M
- Visual binaries
- Detached OB systems
- Resolved spectroscopic binaries