What can we learn about stars from their spectra?

[Graph showing the relationship between wavelength and flux for different temperatures (T=10,000 K, T=8000 K, T=5800 K, T=3000 K). The y-axis represents flux per Angstrom, and the x-axis represents wavelength in Angstroms.]
What can we learn about stars from their spectra?

Observed Spectra of
Vega-type Star
Solar-type Star

Temperature:
- T=10,000 K
- T=8000 K
- T=5800 K
- T=3000 K
What can we learn about stars from their spectra?

Here Stars Look almost exactly like blackbodies

Observed Spectra of
Vega-type Star
Solar-type Star

T=10,000 K
T=8000 K
T=5800 K
T=3000 K
What can we learn about stars from their spectra?

Observed Spectra of
Vega-type Star
Solar-type Star

Lots of absorption from atoms in the stars' atmospheres

Here Stars Look almost exactly like blackbodies

T=10,000 K
T=8000 K
T=5800 K
T=3000 K
Spectral Lines

Joseph von Fraunhofer (1787-1826)
Spectral Lines

Joseph von Fraunhofer (1787-1826)

Identified “black” lines in Sunlight when dispersed by a prism. Regions with no emission. By 1814, Fraunhofer had cataloged over 475 of these lines.
Spectral Lines: fingerprints of atoms!!!

• Fraunhofer identified one line in the Solar spectrum to correspond to a line of wavelength 590 nm observed when salt is burnt in a flame. Corresponds to Sodium in the Sun’s atmosphere.

• Studies of absorption lines in the Sun’s spectrum identified “Helium” (from Greek Helios for Sun) in 1868, not seen previously on Earth (and not discovered until 1895).

• Other studies of Sun showed that it contains a wide variety of elements.

• Other stars show similar (yet often different) absorption lines.
Fraunhofer Lines

<table>
<thead>
<tr>
<th>Wavelength [nm]</th>
<th>Name</th>
<th>Atom</th>
</tr>
</thead>
<tbody>
<tr>
<td>393.368</td>
<td>K</td>
<td>Ca^{+}</td>
</tr>
<tr>
<td>396.849</td>
<td>H</td>
<td>Ca^{+}</td>
</tr>
<tr>
<td>410.175</td>
<td>h (H\delta)</td>
<td>H^{0}</td>
</tr>
<tr>
<td>422.674</td>
<td>g</td>
<td>Ca^{0}</td>
</tr>
<tr>
<td>434.048</td>
<td>G, H\gamma</td>
<td>H^{0}</td>
</tr>
<tr>
<td>438.355</td>
<td>e</td>
<td>Fe^{0}</td>
</tr>
<tr>
<td>438.356</td>
<td>d</td>
<td>Fe^{0}</td>
</tr>
<tr>
<td>486.134</td>
<td>F (H\beta)</td>
<td>H^{0}</td>
</tr>
<tr>
<td>495.761</td>
<td>c</td>
<td>Fe^{0}</td>
</tr>
<tr>
<td>516.733</td>
<td>b4</td>
<td>Mg^{0}</td>
</tr>
<tr>
<td>517.270</td>
<td>b2</td>
<td>Mg^{0}</td>
</tr>
<tr>
<td>518.362</td>
<td>b1</td>
<td>Mg^{0}</td>
</tr>
<tr>
<td>527.039</td>
<td>E</td>
<td>Fe^{0}</td>
</tr>
<tr>
<td>588.997</td>
<td>D2</td>
<td>Na^{0}</td>
</tr>
<tr>
<td>589.594</td>
<td>D1</td>
<td>Na^{0}</td>
</tr>
<tr>
<td>627.661</td>
<td>a</td>
<td>O_{2}</td>
</tr>
<tr>
<td>656.281</td>
<td>C (H\alpha)</td>
<td>H^{0}</td>
</tr>
<tr>
<td>686.719</td>
<td>B</td>
<td>O_{2}</td>
</tr>
<tr>
<td>759.370</td>
<td>A</td>
<td>O_{2}</td>
</tr>
</tbody>
</table>

The diagram on the left shows the visible spectrum, with lines marked at various wavelengths. The table on the right lists the wavelengths, names, and corresponding atoms for the Fraunhofer lines.
Spectral Lines

Intensity (counts)

Wavelength (nanometers)
Analyzing Absorption Spectra

• Each element produces a specific set of absorption (and emission) lines.

• Comparing the relative strengths of these sets of lines, we can study the composition of gases.

### Table 7-2
The Most Abundant Elements in the Sun

<table>
<thead>
<tr>
<th>Element</th>
<th>Percentage by Number of Atoms</th>
<th>Percentage by Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen</td>
<td>91.0</td>
<td>70.9</td>
</tr>
<tr>
<td>Helium</td>
<td>8.9</td>
<td>27.4</td>
</tr>
<tr>
<td>Carbon</td>
<td>0.03</td>
<td>0.3</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>0.008</td>
<td>0.1</td>
</tr>
<tr>
<td>Oxygen</td>
<td>0.07</td>
<td>0.8</td>
</tr>
<tr>
<td>Neon</td>
<td>0.01</td>
<td>0.2</td>
</tr>
<tr>
<td>Magnesium</td>
<td>0.003</td>
<td>0.06</td>
</tr>
<tr>
<td>Silicon</td>
<td>0.003</td>
<td>0.07</td>
</tr>
<tr>
<td>Sulfur</td>
<td>0.002</td>
<td>0.04</td>
</tr>
<tr>
<td>Iron</td>
<td>0.003</td>
<td>0.1</td>
</tr>
</tbody>
</table>

By far the most abundant elements in the Universe
1859: Kirchhoff “explains” spectra of stars
The Spectra of Stars

Inner, dense layers of a star produce a continuous (blackbody) spectrum.

Cooler surface layers absorb light at specific frequencies.

=> Spectra of stars are absorption spectra.
Kirchhoff’s Laws of Radiation (1)

1. A solid, liquid, or dense gas excited to emit light will radiate at all wavelengths and thus produce a continuous spectrum.

No trace of individual atoms in this thermal spectrum! Depends only on temperature.
Kirchhoff’s Laws of Radiation (2)

3. If light comprising a continuous spectrum passes through a cool, low-density gas, the result will be an absorption spectrum.

- Light excites electrons in atoms to higher energy states.
- Frequencies corresponding to the transition energies are absorbed from the continuous spectrum.
Kirchhoff’s Laws of Radiation (3)

2. A low-density gas excited to emit light will do so at specific wavelengths and thus produce an emission spectrum.

Transition back to lower states emits light at specific frequencies

Light excites electrons in atoms to higher energy states
Spectrographs, work on principle of Diffraction

\[ d \sin \theta = \begin{cases} 
  n\lambda & (n=0,1,2,3...) \text{, constructive interference} \\
  (n-1/2)\lambda & (n=0,1,2,3...) \text{, destructive interference}
\end{cases} \]

Double-Slit Experiment of Thomas Young (1773-1829)

- \( n \) = the “order” of the spectrum
- Ability to resolve two wavelengths, separated by \( \Delta \lambda = |\lambda_1 - \lambda_2| \) is
  \[ \Delta \lambda = \lambda / (n \ N) \]
  where \( N = \) number of lines

\[ \lambda / \Delta \lambda = n \times N = \text{“Resolving Power”} = R \]
“Continuous” Spectra

Flux per Angstrom

Wavelength [Å]

T=10,000 K
T=8000 K
T=5800 K
T=3000 K
“Continuous” Spectra

Observed Spectra of
Vega-type Star
Solar-type Star

Wavelength [Å]

Flux per Angstrom

T=10,000 K
T=8000 K
T=5800 K
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“Continuous” Spectra

T=10,000 K
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"Continuous" Spectra

Observed Spectra of Vega-type Star
Solar-type Star

T=10,000 K
T=8000 K
T=5800 K
T=3000 K

Here Stars Look almost exactly like blackbodies

Lots of absorption from atoms in the stars’ atmospheres (more next week)
Emission Line Galaxy

PN G000.2+06.1

$F_{\lambda} [10^{-14} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ Å}^{-1}]$

Wavelength (Å)
Galaxy with Absorption and Emission Lines
When dense Hydrogen gas is heated, it shows emission lines at exact wavelengths. By 1885, 14 spectral lines in Hydrogen had been measured. After much trial and error, Johann Balmer (1825-1898) found that the wavelengths of hydrogen corresponded to a pattern, now called the **Balmer series** made of the **Balmer lines**.

\[
\frac{1}{\lambda} = R_H \left( \frac{1}{4} - \frac{1}{n^2} \right), \quad n=3, 4, 5, \ldots, \infty
\]

\[R_H = 1.09677583 \times 10^7 \text{ m}^{-1}\] is an experimentally measured quantity called the Rydberg constant for hydrogen.
Hydrogen lines

Balmer realized that he could generalize this to:

\[
\frac{1}{\lambda} = R_H \left( \frac{1}{m^2} - \frac{1}{n^2} \right), \quad m=2, n=3, 4, 5, ..., \infty
\]

And he predicted that other line series could be found with \( m < n \) (both integers).

In 1906, Theodore Lyman confirmed the series for \( m=1, n=2, 3, 4, ... \) as the Lyman Series.

In 1908, Friedrich Paschen confirmed the series for \( m=3, n=4, 5, 6, ... \) as the Paschen Series.

<table>
<thead>
<tr>
<th>Series Name</th>
<th>Symbol</th>
<th>n to m</th>
<th>Wavelength [nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lyman</td>
<td>Ly(\alpha)</td>
<td>2 to 1</td>
<td>121.567</td>
</tr>
<tr>
<td></td>
<td>Ly(\beta)</td>
<td>3 to 1</td>
<td>102.572</td>
</tr>
<tr>
<td></td>
<td>Ly(\gamma)</td>
<td>4 to 1</td>
<td>97.254</td>
</tr>
<tr>
<td></td>
<td>Ly limit</td>
<td>(\infty) to 1</td>
<td>91.18</td>
</tr>
<tr>
<td>Balmer</td>
<td>H(\alpha)</td>
<td>3 to 2</td>
<td>656.281</td>
</tr>
<tr>
<td></td>
<td>H(\beta)</td>
<td>4 to 2</td>
<td>486.132</td>
</tr>
<tr>
<td></td>
<td>H(\gamma)</td>
<td>5 to 2</td>
<td>434.048</td>
</tr>
<tr>
<td></td>
<td>H limit</td>
<td>(\infty) to 2</td>
<td>364.6</td>
</tr>
<tr>
<td>Paschen</td>
<td>Pa(\alpha)</td>
<td>4 to 3</td>
<td>1875.10</td>
</tr>
<tr>
<td></td>
<td>Pa(\beta)</td>
<td>5 to 3</td>
<td>1281.81</td>
</tr>
<tr>
<td></td>
<td>Pa(\gamma)</td>
<td>6 to 3</td>
<td>1093.81</td>
</tr>
<tr>
<td></td>
<td>Pa limit</td>
<td>(\infty) to 3</td>
<td>820.4</td>
</tr>
</tbody>
</table>
"Planetary" model of atom

Proton mass: $1.7 \times 10^{-27}$ kg
Electron mass: $9 \times 10^{-31}$ kg

where the mass of the electron is 1/2000 the mass of the proton
and the mass of the proton equals the mass of the neutron

A classical bound charge emits or absorbs continuous spectrum
Catastrophe with atoms

Accelerating electron produces EM radiation (light), loses energy and spirals into nucleus in ~ 1 ns, i.e. atoms should be unstable!

According to classical physics, an electron in orbit around an atomic nucleus should emit electromagnetic radiation (photons) continuously, because it is continually accelerating in a curved path. The resulting loss of energy implies that the electron should spiral into the nucleus in a very short time (i.e. atoms cannot exist).
Quantum Mechanics I: wave-particle nature of light

1900: light quanta postulated by Planck in order to explain his formula for black-body spectrum

1905: Quantum theory of light proposed by Einstein to explain the mysteries of the photoeffect
Photo-electric Effect

Light (photons). Could have different brightness (# of photons per s) or photons of different frequencies

Electrons observed with Kinetic energy up to some measured maximum, $K_{\text{max}}$
Photo-electric Effect

Light (photons). Could have different brightness (# of photons per s) or photons of different frequencies

Electrons observed with Kinetic energy up to some measured maximum, $K_{\text{max}}$

Observations showed that $K_{\text{max}}$ does not depend on the brightness (intensity) of light.
Photo-electric Effect

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Observations showed that $K_{\text{max}}$ does not depend on the brightness (intensity) of light.

But, observations showed that $K_{\text{max}}$ does depend on the frequency of the light.
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Each metal has an observed cutoff frequency, such that electrons only released when $\nu > \nu_{\text{cut}}$. 
Photo-electric Effect

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Electrons observed with Kinetic energy up to some measured maximum, $K_{\text{max}}$

Observations showed that $K_{\text{max}}$ does not depend on the brightness (intensity) of light.

But, observations showed that $K_{\text{max}}$ does depend on the frequency of the light.

Classically, amount of energy carried by light is $S = (1/\mu_0) |E \times B|$. No dependence on frequency.

Each metal has an observed cutoff frequency, such that electrons only released when $\nu > \nu_{\text{cut}}$. 
Photo-electric Effect: explanation

Einstein knew of Planck’s work. He postulated that light stream contained particles (photons), each of which has some energy, \( E = h \nu = hc/\lambda \).

When photon hits metal, energy may be absorbed by an electron, which may become “unbound” from metal and released. The minimum binding energy of electrons in the metal is called its **Work function**, which gives the equation for \( K_{\text{max}} \):

\[
K_{\text{max}} = E - \phi = h \nu - \phi = hc/\lambda - \phi.
\]

For \( K_{\text{max}} = 0 \), \( \nu_{\text{cut}} = \phi/h \).

This is the cutoff frequency.
Compton Scattering: another proof of quantum nature of light

Compton studied experiments of collisions between X-rays (photons) and electrons. Collisions changed the frequency of the photons and scattered the electrons.

Energy and Momentum conserved.

$$\Delta \lambda = \lambda_f - \lambda_i = \left(\frac{h}{m_e c}\right)(1 - \cos \theta)$$

Proved that photons are massless, but carry momentum. Known as Compton Effect.

$$\lambda_C = \frac{h}{m_e c}$$ is the Compton Wavelength
Quantum mechanics II: wave-particle nature of matter
What does an atom look like?

1904: J.J. Thomson’s plum pudding model of an atom
Rutherford Experiment

Gold Foil

Detecting screen

Slit

∞ particle emitter

Ernest Rutherford (1871-1937)
Rutherford Experiment

Old Atomic Model

Rutherford’s Atomic Model
Classical (incorrect) picture of atoms

As electron “accelerates” around nucleus, it emits radiation, loses energy and spirals into nucleus. The frequency of the radiation increases to infinity as $R$ goes to zero. For hydrogen, the electron should collide with the nucleus in less than one-billionth of a second.
Bohr model of Atom

Niels Bohr
(1885-1962)
proposed new atomic model in 1913.

1. Electron in an atom can occupy one of a discrete set of states (quantized orbits) with discrete energies and radii. There are no states “in between”. The electron can make a discontinuous transition from one stationary state to another. It emits or absorbs radiation during these jumps.

2. There is a stable orbit (ground state) on which electrons do not radiate.

3. When an electron makes a transition, the energy difference is released as a single photon of frequency, $\nu = E/h$.

4. Permitted orbits are characterized by quantized values of the orbital angular momentum with $L = n \hbar / 2\pi = n \hbar$ ($\hbar$ is “h bar”).
Bohr model of Atom

Coulomb’s Law

\[
\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q_1q_2}{r^2} \hat{r}
\]

Proton has charge of \( q = +e = 1.602176462 \times 10^{-19} \text{ C} \)

Electron has charge of \( q = -e \)

This is a two-body problem, for which we can model as a reduced mass, \( \mu \), orbiting a total mass \( M \).

For this problem:

\[
\mu = \frac{m_em_p}{m_e + m_p} = \frac{m_e(1836.15266m_e)}{m_e + 1836.15266m_e} = 0.999455679m_e
\]

\[
M = m_e + m_p = \left( m_p/1836.15266 \right) + m_p = 1.0005446m_p
\]

\[
E = -K = -\frac{1}{2} \mu v^2 = -\frac{1}{8\pi\varepsilon_0} \frac{e^2}{r}
\]
Bohr model of Atom

\[ E = -K = -\frac{1}{2} \mu v^2 = -\frac{1}{8\pi \varepsilon_0} \frac{e^2}{r} \]

Bohr’s quantization of angular momentum gives

\[ L = \mu vr = n\hbar \]

Rewrite top equation in terms of angular momentum and then solve for \( r \):

\[ \frac{1}{2} \frac{e^2}{8\pi \varepsilon_0} \frac{1}{r} = \frac{1}{2} \mu v^2 = \frac{1}{2} \frac{\left(\mu vr\right)^2}{\mu r^2} = \frac{1}{2} \frac{\left(n\hbar\right)^2}{\mu r^2} \]

\[ r = \frac{1}{2} \frac{\left(n\hbar\right)^2}{\mu} \times \frac{8\pi \varepsilon_0}{e^2} \]

\[ r_n = \frac{4\pi \varepsilon_0 \hbar^2}{\mu e^2} n^2 = a_0 n^2 \]

\( a_0 = \) Bohr Radius = 0.0529 nm

Insert eqn. for \( r_n \) into equation for Energy gives:

Rydberg energy (binding energy):

\[ R_y = \frac{m_e e^4}{32\pi^2 \varepsilon_0^2 \hbar^2} = \frac{m_e e^4}{2\hbar^2} \text{ [CGS]} \approx 13.6 \text{ eV} \]

\[ E_n = -\frac{1}{8\pi \varepsilon_0} \frac{e^2}{r_n} = -R_y \frac{1}{n^2} \]

\( n \) is the principal quantum number

\[ 1 \text{ eV} = e \times 1 \text{ V} = 1.602 \times 10^{-19} \text{ J} \]
Emission Lines in Hydrogen

\[ E_{\text{photon}} = E_{\text{high}} - E_{\text{low}} \]

\[ E_{n=3} - E_{n=2} = -1.50 \text{ eV} - (-3.40 \text{ eV}) = 1.90 \text{ eV} \]

Wavelength of that photon is \( E = \frac{hc}{\lambda} \), or \( \lambda = \frac{hc}{E} = 656.3 \text{ nm} \).
Kirchhoff’s Laws:

- A hot, dense gas or hot solid object produces a continuous Spectrum with no dark spectral lines.

- A hot, diffuse (low density) gas produces bright spectral lines (emission lines).

- A cool, diffuse gas in front of a source of continuous spectrum produces dark spectral lines (absorption lines) in the continuous spectrum.
Kirchhoff’s Laws, Restated.

• A hot, dense gas or hot solid object produces a continuous spectrum with no dark spectral lines. This is blackbody radiation emitted at any temperature, $T > 0$ K, with a spectrum described by the Planck function $B_\lambda(T)$ and $B_\nu(T)$. The wavelength at which the Planck Function $B_\lambda(T)$ reaches its maximum is given by Wien’s displacement law.

• A hot, diffuse (low density) gas produces bright emission lines. Emission lines are produced when an electron makes a downward transition from a higher orbit to a lower orbit. The energy lost by then electron is carried away by a single photon.

• A cool, diffuse gas in front a source of continuous spectrum produces dark absorption lines in the continuous spectrum. Absorption lines are produced when an electron absorbs a photon with enough energy equal to the energy difference between the two transitions.
Transition to Quantum Mechanical Picture of Atoms

Bohr’s model was successful, but did not account for all observed properties of atoms. This was a “semi-classical” picture of atoms as mini-solar systems. Quantized orbits were postulated. Full picture needed quantum mechanics.

The explanation of quantized orbits arrived from the Ph.D. thesis of Louis de Broglie in 1923. de Broglie argued that since light can display wave and particle properties, then perhaps matter can also be a particle and a wave too.

Energy and momentum of a particle are related to its de Broglie wavelength:

\[ p = \hbar k = \frac{2\pi \hbar}{\lambda} = \frac{h}{\lambda} \]

\[ \lambda = \frac{h}{p} = \frac{h}{mv} \]

\[ E = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m} \]
Transition to Quantum Mechanical Picture of Atoms

Examples:
\[ \lambda = \frac{h}{p} = \frac{h}{(m \, v)} \]

1. electron moving at 1% the speed of light (3x10^6 m/s):

\[ \lambda = \frac{h}{(m_e \, v)} = \frac{(6.63 \times 10^{-34} \text{ J s})}{(9.11 \times 10^{-31} \text{ kg} \times 3 \times 10^6 \text{ m/s})} = 2.42 \times 10^{-10} \text{ m} = 0.242 \text{ nm} \]

2. jogging person with m = 80 kg and v = 2 m/s:

\[ \lambda = \frac{h}{(m \, v)} = \frac{(6.63 \times 10^{-34} \text{ J s})}{(80 \text{ kg} \times 2 \text{ m/s})} = 4.14 \times 10^{-36} \text{ m} = 4.14 \times 10^{-27} \text{ nm} \]

Roughly 1 billionth of a billionth of the size of the nucleus of an atom!
Interference and diffraction of electron waves
Why the orbits are quantized

An electron orbiting a proton with a circumference equal to 3 de Broglie wavelengths.

In atoms, electrons *must* orbit with integral number of wavelengths $n\lambda$ where $\lambda$ is the de Broglie wavelength. Else, the electron wave will be out of phase with itself and destructively interfere.
Heisenberg’s Uncertainty principle

One wave, well known wavelength, can determine momentum exactly by, \( p = \frac{h}{\lambda} \).

But, because wave has infinite number of peaks, \( x \) location is unknown.

Superposition of many waves with a combination of wavelengths.

\[
\Psi = \sum_{n} \sin\left(\frac{2\pi x}{(n\lambda)}\right)
\]

\( \lambda \) not exactly known, \( p \) has uncertainty. But, \( x \) can be determined more accurately.
Transition to Quantum Mechanical Picture of Atoms

Heisenberg’s Uncertainty principle.

formally: $\Delta x \Delta p \geq \frac{\hbar}{2}$  $\Delta E \Delta t \geq \frac{\hbar}{2}$

in practice: $\Delta x \Delta p \approx \hbar$  $\Delta E \Delta t \approx \hbar$

Most formulas of quantum mechanics can be derived (within a factor of 2) using classical mechanics and the uncertainty principle
If an electron is a wave around the atom, instead of a particle in orbit `where' is the electron at any particular moment?

The answer is that the electron can be anywhere around the atom. But 'where' is not evenly distributed. The electron as a wave has a maximum chance of being observed where the wave has the highest amplitude. Thus, the electron has the highest probability to exist at a certain orbit.

Werner Heisenberg

1920s

Erwin Shrödinger
Atom is mostly empty space!

Size of proton or neutron: $\sim 10^{-15}$ m

Size of an electron cloud: $\sim 10^{-10}$ m (1 Angstrom)

Proton mass: $1.7 \times 10^{-27}$ kg
Electron mass: $9 \times 10^{-31}$ kg
Transition to Quantum Mechanical Picture of Atoms

Quantum Numbers

\( n \): orbital state, determines energy

\( \ell \): angular momentum state. Comes from true quantization of angular momentum, \( L = \sqrt{\ell (\ell +1)} \hbar \), where \( \ell =0,1,2,...,n-1 \). Historically, spectroscopic designations are s,p,d,f,g,h corresponding to \( \ell =0,1,2,3,4,5 \)

\( m_\ell \): z-component of angular momentum. Seen when electric fields applied to atoms. Allowed values are all integers from \(-\ell\) to \(+\ell\).

\( m_s \): “Spin”. Intrinsic angular momentum of particles. Two types of particles. Fermions (electrons, protons, neutrons) with \( m_s = \pm \frac{1}{2} \). Bosons (photons) with \( m_s = \text{integers} \).
Quantum Numbers: \( n, \ell, m_\ell, m_s \)

No two Fermions can have all the same quantum \# values. (This is important for atoms, but also for the structure of white dwarfs and neutron stars.)

Selection Rules for atomic transitions:
Allowed transitions with \( \Delta \ell = \pm 1 \).

Allowed transitions occur on \( 10^{-8} \) s timescales.

Transitions with \( \Delta \ell \neq \pm 1 \) called “Forbidden” transitions. These do occur from metastable atomic states, but over a much longer time.

Allowed transitions for Hydrogen
Some applications of spectral line measurements in astronomy

- Measuring chemical composition
- Stellar classification
- Measurement of line-of-sight velocities using Doppler effect
Spectral Lines

Stellar Classification

O6.5
B0
B6
A1
A5
F0
F5
G0
G5
K0
K5
M0
M5

Ca  H  H  Fe  Na  H
# Spectral Classification of Stars

## Mnemonics to remember the spectral sequence:

<table>
<thead>
<tr>
<th>Spectral Class</th>
<th>Approximate Temperature (K)</th>
<th>Hydrogen Balmer Lines</th>
<th>Other Spectral Features</th>
<th>Naked-Eye Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>40,000</td>
<td>Weak</td>
<td>Ionized helium</td>
<td>Meissa (O8)</td>
</tr>
<tr>
<td>B</td>
<td>20,000</td>
<td>Medium</td>
<td>Neutral helium</td>
<td>Achernar (B3)</td>
</tr>
<tr>
<td>A</td>
<td>10,000</td>
<td>Strong</td>
<td>Ionized calcium weak</td>
<td>Sirius (A1)</td>
</tr>
<tr>
<td>F</td>
<td>7,500</td>
<td>Medium</td>
<td>Ionized calcium weak</td>
<td>Canopus (F0)</td>
</tr>
<tr>
<td>G</td>
<td>5,500</td>
<td>Weak</td>
<td>Ionized calcium medium</td>
<td>Sun (G2)</td>
</tr>
<tr>
<td>K</td>
<td>4,500</td>
<td>Very weak</td>
<td>Ionized calcium strong</td>
<td>Arcturus (K2)</td>
</tr>
<tr>
<td>M</td>
<td>3,000</td>
<td>Very weak</td>
<td>TiO strong</td>
<td>Betelgeuse (M2)</td>
</tr>
</tbody>
</table>

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Stellar Spectra

Surface temperature
The Doppler effect: apparent change in the wavelength or frequency of radiation caused by motion of the source.

$$\frac{V_{\text{radial}}}{c} = \frac{\Delta \lambda}{\lambda_0} = -\frac{\Delta f}{f_0}$$; \[ \frac{V_{\text{radial}}}{c} << 1 \]
The Doppler Effect

The light of a moving source is blue/red shifted by

\[
\frac{\Delta \lambda}{\lambda_0} = \frac{v_r}{c}
\]

\(\lambda_0\) = actual wavelength emitted by the source

\(\Delta \lambda\) = Wavelength change due to Doppler effect

\(v_r\) = radial velocity (along the line of sight)
The Doppler effect: apparent change in the wavelength of radiation caused by the motion of the source

Shift \( z = \frac{(\text{Observed wavelength} - \text{Rest wavelength})}{(\text{Rest wavelength})} \)

Doppler effect:

\[ z = \frac{\Delta \lambda}{\lambda_0} = \frac{V_{\text{rad}}}{c} \; ; \quad V_{\text{rad}} \ll c \]
The Doppler Effect

The Doppler effect allows us to measure the source’s radial velocity.

\[ \frac{\Delta \lambda}{\lambda_0} = \frac{v_r}{c} \]

If (proper) transverse motion is detectable and distance is known, we can calculate the transverse and the total velocity.
Doppler effect

Applications: Measure line of sight velocities, gravitational effects, gas temperatures

Comparing Sun’s spectrum to other stars. For non-relativistic speeds, the line-of-sight (called the *recessional*) velocity is

\[
\frac{\lambda_{\text{obs}} - \lambda_{\text{rest}}}{\lambda_{\text{rest}}} = \frac{\Delta \lambda_{\text{obs}}}{\lambda_{\text{rest}}} = \frac{v_r}{c}
\]

In Vega, important Hydrogen line has \(\lambda_{\text{obs}} = 656.251\) nm compared to \(\lambda_{\text{rest}} = 656.281\) nm, measured in the lab.

\[
v_r = c \frac{\lambda_{\text{obs}} - \lambda_{\text{rest}}}{\lambda_{\text{rest}}} = -13.9 \text{ km s}^{-1}
\]

Vega also has a measured proper motion (perpendicular to line-of-sight) of \(\mu=0.35077''\) yr\(^{-1}\). At \(r=7.76\) pc, the proper motion (or transverse velocity) is \(v_\theta = r\mu = 12.9\) km s\(^{-1}\). The total velocity is

\[
v = \sqrt{v_r^2 + v_\theta^2} = 19.0 \text{ km s}^{-1}
\]