PHYSICS 218 Exam 2
Fall, 2006 STEPS

Do not fill out the information below until instructed to do so!

Name: ________________________
Signature: ____________________
Student ID: __________________
E-mail: _______________________
Section Number: _______________

- No calculators are allowed in the test.
- Be sure to put a box around your final answers and clearly indicate your work to your grader.
- All work must be shown to get credit for the answer marked. If the answer marked does not obviously follow from the shown work, even if the answer is correct, you will not get credit for the answer.
- Clearly erase any unwanted marks. No credit will be given if we can’t figure out which answer you are choosing, or which answer you want us to consider.
- Partial credit can be given only if your work is clearly explained and labeled.

Put your initials here after reading the above instructions:

Mathematical expressions:

If \( f(x) = kx^n \)

\[
\frac{df(x)}{dx} = nkx^{n-1}
\]

\[
\int f(x)dx = \frac{1}{n+1}kx^{n+1} + C
\]

\[
\int_a^b f(x)dx = \frac{1}{n+1}kb^{n+1} - \frac{1}{n+1}ka^{n+1}
\]

For grader use only:
Problem 1 (30) _________
Problem 2 (40) _________
Problem 3 (30) _________
Total (100) ___________
Problem 1: (30 points)

Two boxes of masses $M_1$ and $m_2$ are placed on an inclined plane of angle $\theta$ lying one on top of the other and connected by a massless unstretchable string attached through a massless frictionless pulley as shown. There is friction between the boxes with coefficient $\mu$ but not between box $M_1$ and the inclined plane. ($M_1$ is greater than $m_2$)

a) (10) If $M_1$ begins to slide down, draw the free-body force diagram for each of the boxes.

b) (10) If $M_1$ begins to slide down, what is the magnitude of its acceleration?

c) (10) What is the minimum coefficient of friction needed for the boxes to remain at rest.
Problem 2: (40 points)

A block of mass \( m \) is moving at a velocity of magnitude \( v_0 \) along a frictionless horizontal surface towards a spring which exerts a force given by \( \vec{F}_{spring} = (-kx - \alpha x^3)i \). The mass of the spring is negligible.

a) (15) Calculate the potential energy function of the force of the spring.

b) (25) Obtain the equation to find the maximum distance the spring will be compressed. Give your answers in terms of \( m \), \( v_0 \), \( k \), \( \alpha \).
Problem 3: (30 points)
A box of mass $M$ starts from rest and it is placed touching a spring with a constant $k$ compressed a distance $A$ from its initial uncompressed length. The mass is released and goes up the inclined plane at angle $\theta$.

a) (15) Determine the maximum height, $h$, it will reach if you assume no friction.

b) (15) If instead of $h$ it reaches a height $h'$, determine the coefficient of friction $\mu$ between the box and the plane.
Physics 218: Mechanics Exam 2, 23 October 2007

Print your name **neatly**:

Last name: 
First name: 

_Sign your name:_ 

Your instructor: ___________ Your section: ___________

Please fill in your Student ID number (UIN): ____-____-____-____-

**IMPORTANT:** Read these directions carefully:

- There are 4 problems totalling 100 points. Check your exam to make sure you have all the pages. Work each problem on the page the problem is on. You may use the back. If you need extra pages, I have plenty up front.

- **Indicate what you are doing!** We cannot give full credit for merely writing down the answer. **Neatness counts!** I will give generous partial credit if I can tell that you are on the right track. This means you must be _neat_ and organized.

**USEFUL INFORMATION**

\[
If \quad f(x) = kx^n \quad \frac{df}{dx} = nkx^{n-1}
\]

\[
If \quad f(x) = kx^n \quad \int_A^B f(x)dx = \frac{1}{n+1}k(B^{n+1} - A^{n+1})
\]

\[
\int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_{\text{tot}} \cdot d\vec{r} = \frac{1}{2}mv^2(\vec{r}_2) - \frac{1}{2}mv^2(\vec{r}_1)
\]

If \( \vec{F} \) is conservative:

\[
\int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} = -[U(\vec{r}_2) - U(\vec{r}_1)]
\]

and

\[
F_x = -\frac{\partial U}{\partial x} \quad F_y = -\frac{\partial U}{\partial y}
\]

**DO NOT WASTE TIME DOING ARITHMETIC**

Grading: 
Problem 1 Problem 2 Problem 3 Problem 4 TOTAL
1. (25 points) Block 1, of mass $m_1$ is placed at rest on an inclined plane. It is attached by a massless, unstretchable string to block 2, of mass $m_2$. The pulley is massless and frictionless and just changes the direction of the tension in the string. Assume the variables are such that $m_1$ slides down the plane, starting at $t = 0$. The coefficient of friction between the plane and $m_1$ is the constant $\mu$.

a. Draw the free body diagrams for block 1 and block 2.

![Free Body Diagram](image)

b. Find the acceleration of block 1.

c. Find the velocity of block 1 as a function of time.
2. (25 points) This is a one-dimensional problem. You need not concern yourself with the y direction. Do not spend very much time on algebra. Once you have one equation with one unknown you should stop!

An object of mass $m$ is placed at the point $x = A$ on a horizontal table and given a velocity of magnitude $v_1$ to the right. The object is attracted to the origin by some mysterious force which has magnitude $\frac{\alpha}{x^2}$ where $\alpha$ is a constant. The coefficient of friction between the table and the object is $\mu$.

\[
\begin{array}{c}
\vec{u}_1 \\
\includegraphics[width=0.5\textwidth]{diagram.png} \\
\end{array}
\]

a. How far will the object go before it turns around and begins to move to the left?

b. How fast will the object be going when it is again at the point $x = A$?
3. (25 points) A block of mass \( m \) is placed on a frictionless table where there is a spring, with spring constant \( k \). The spring is not stretched or compressed at the point \( x = 0 \). The block is pushed to the left, so that the spring is compressed an amount \( A \), and released from rest. Besides the normal force, gravity and the spring there is another force acting on the block given by \( \vec{F}_1 = c_1 \vec{i} + c_2 \vec{j} \) where \( c_1 \) and \( c_2 \) are known constants, and \( c_2 < mg \).

a. How fast will the block be going at \( x = 0 \)?

b. How fast will the block be going at \( x = B \)? (Remember the spring is not attached to the block!)

c. Assuming the block makes it up the frictionless incline to the point D, how fast will it be going at the point D where \( x = L \) and \( y = H \)?
4. (25 points) This is a one-dimensional problem. You need not concern yourself with the y direction. An object of mass $m$ is acted upon by a force given by

$$F_x = \frac{\alpha}{x^3} - \frac{\beta}{x^2}$$

where $\alpha$ and $\beta$ are positive constants.

a. Verify that this force is conservative by finding a potential energy function.

b. If the object is placed at the point $x = \frac{\alpha}{\beta}$ and given a velocity of magnitude $v_1$ in the positive $x$ direction, what will its velocity be at the point $x = A$?
USEFUL INFORMATION

If \( f(x) = kx^n \) \( \frac{df}{dx} = nkx^{n-1} \)

If \( f(x) = kx^n \) \( \int_A^B f(x) \, dx = \frac{1}{n+1} k(B^{n+1} - A^{n+1}) \)

If \( f(x) = kx^n \) \( \int f(x) \, dx = \frac{1}{n+1} kx^{n+1} + C \)

\[ \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_{\text{tot}} \cdot d\vec{r} = \frac{1}{2} m v^2(\vec{r}_2) - \frac{1}{2} m v^2(\vec{r}_1) \]

If \( \vec{F} \) is conservative:

\[ \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} = -[U(\vec{r}_2) - U(\vec{r}_1)] \]

and

\[ F_x = -\frac{\partial U}{\partial x} \quad F_y = -\frac{\partial U}{\partial y} \]

DO NOT WASTE TIME DOING ARITHMETIC
1. (25 points) A box of mass $m$ is on an inclined plane with coefficient of friction $\mu$ between the box and the plane. A man is trying to keep the box from moving by pushing on the box with a force parallel to the plane. The box starts from rest and slides down the plane to the point a distance $B$ from the top and then gets pushed back to the top where it is again at rest.

![Diagram of box on inclined plane]

a. How much work is done by the force of gravity for the total distance the box moves?

b. How much work is done by the force of friction for the total distance the box moves?

c. How much work is done by the force exerted by the man for the total distance the box moves?
2. (25 points) An object of mass \( m \) is placed at the point \( x = A \) on a horizontal table and given, at \( t = 0 \), a velocity of magnitude \( v_1 \) to the right. The coefficient of friction between the table and the object is \( \mu \). (NO ALGEBRA PLEASE!)

\[ x = A \]

\[ \vec{t} \]

\[ \rightarrow_{+x} \]

a. (5 points) Where will the object stop if \( \mu \) has the constant value \( \mu_0 \)?

b. (10 points) Where will the object stop if \( \mu \) is a function of \( x \) given by \( \mu = \mu_0(1 + \frac{x}{L}) \) where \( L \) is a known distance?

c. (10 points) Where will the object stop if \( \mu \) is a function of \( t \) given by \( \mu = \mu_0(1 + \frac{t}{S}) \) where \( S \) is a known time?
3. (25 points) Two unequal masses are connected by a massless, unstretchable string which goes over a frictionless pulley.

The lighter mass, $m_1$ is attached to a spring which is also attached to the floor. The spring has spring constant $k$ and is unstretched when the masses are held at rest in the positions shown. The masses are then released.

a. Isolate $m_2$ and draw the free body diagram for it.

b. Isolate $m_1$ and draw the free body diagram for it.

c. Apply the Work Energy Theorem to each mass if the heavier mass moves down a distance $H$.

d. Find the velocity of the heavier mass when it has moved down a distance $H$. 
4. (25 points) This is a one-dimensional problem. You need not concern yourself with the y direction. An object of mass m is acted upon by a force given by

$$F_x = -\beta(x - c)$$

where c and \( \beta \) are positive constants.

a. Determine whether or not this force is conservative.

b. If the object is placed at the point \( x = A \) and given a velocity of magnitude \( v_1 \) in the positive x direction, what will its velocity be at the point \( x = \frac{A}{2} \) if the above force is the only force acting on the object?

\[ \frac{A}{2} \quad x = A \rightarrow x \]

c. Where will the kinetic energy have its maximum value?
USEFUL INFORMATION

If \( f(x) = kx^n \), \[ \frac{df}{dx} = nkx^{n-1} \]

If \( f(x) = kx^n \), \[ \int_A^B f(x)dx = \frac{1}{n+1}k(B^{n+1} - A^{n+1}) \]

If \( f(x) = kx^n \), \[ \int f(x)dx = \frac{1}{n+1}kx^{n+1} + C \]

If \( \vec{F} \) is conservative:
\[ \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_{\text{tot}} \cdot d\vec{r} = \frac{1}{2}mv^2(\vec{r}_2) - \frac{1}{2}mv^2(\vec{r}_1) \]

If \( \vec{F} \) is conservative:
\[ \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} = [U(\vec{r}_2) - U(\vec{r}_1)] \]

and
\[ F_x = -\frac{\partial U}{\partial x} \quad F_y = -\frac{\partial U}{\partial y} \]

DO NOT WASTE TIME DOING ARITHMETIC

1.
2.
3.
4.
1. (25 points) This is a one-dimensional problem. An object of mass $m$ is acted upon by a force given by

$$F_x = -(c_1x^2 - c_2x)$$

where $c_1$ and $c_2$ are positive constants.

a. Determine whether or not this force is conservative.

b. If the object is placed at the point $x = A$ and given a velocity of magnitude $v_1$ in the positive $x$ direction, at what point would the object stop moving in the positive $x$ direction and begin moving in the negative $x$ direction, assuming the above force is the only force acting on the object? (No algebra please!)

\[ \begin{array}{c}
  x = 0 \\
  x = A
\end{array} \rightarrow \rightarrow -x \]

c. Find the object's kinetic energy as a function of $x$, given the conditions in b. above.
2. (25 points) A small block of mass, \( m_1 \), is placed on top of a larger block mass, \( m_2 \), and the top block is connected to a third block, \( m_3 \), by a massless, unstretchable string which goes over a frictionless pulley. (The pulley therefore changes the direction of the force exerted by the string but not its magnitude.) The coefficient of friction between blocks is \( \mu \) and the surface that the large block is on is frictionless. Assume the masses are such that the third block moves down and that the other blocks move together, in other words there is no slipping.

a. Isolate \( m_1 \), draw the free body diagram for it and apply Newton's Second Law.

b. Isolate \( m_2 \), draw the free body diagram for it and apply Newton's Second Law.

c. Isolate \( m_3 \), draw the free body diagram for it and apply Newton's Second Law.

e. What is the acceleration of \( m_3 \) ?

e. What is the largest value that the acceleration of \( m_3 \) can have before slipping occurs?
3. A block of mass $m$ slides down an inclined plane, starting from rest. In addition to the other forces acting on the body there is a mysterious force acting that has a potential energy function given by

$$U(x, y) = \beta(x^2 + y^2)$$

where $\beta$ is a known constant and $x$ and $y$ are defined in the figure.

a. Find $\vec{F}_{mys}$ the mysterious force in terms of $x, y,$ and $\beta$.

b. Find the velocity of the block when it reaches the bottom of the plane with no friction.

c. Find the velocity of the block when it reaches the bottom if there were a coefficient of friction between the block and the plane $\mu$. 
4. (25 points) In a famous Physics 218 experiment it was discovered that a real spring doesn't totally follow Hooke's Law. The force exerted by the spring is found to be a linear function of the amount stretched, as Hooke's Law says. However, instead of having \( F_y = -ky \) where \( y \) is the amount stretched, the actual force exerted by the spring is approximately given by \( F_y = -(ky + b) \). Here \( k \) and \( b \) are known constants. Thus if the spring is vertical there will be no stretching of the spring when a mass \( m \) is hung from it unless \( mg \) is greater than \( b \).

\[
\begin{align*}
\downarrow &+y \\
\text{- - - - - - \text{spring not stretched}} \\
\hline
\text{--- y = 0}
\end{align*}
\]

- a. Given this force, with \( b \) known, determine the equilibrium amount the spring will be stretched if a mass with \( mg > b \) is suspended from it. (Call this \( y_{eq} \) as in the lab.)

- b. Find the maximum value of the spring's extension if such a mass were released from rest from \( y = 0 \), the point where the spring is not stretched. How is this distance related to \( y_{eq} \)?

- c. Find the maximum value of the mass's kinetic energy if it were released from rest from the point \( y = 0 \). (Not too much algebra please.)