These are your formula sheets

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Derivatives:

\[ \frac{d}{dx} ax^n = an x^{n-1} \]
\[ \frac{d}{dx} \sin ax = a \cos ax \]
\[ \frac{d}{dx} \cos ax = -a \sin ax \]
\[ \frac{d}{dx} e^{ax} = ae^{ax} \]
\[ \frac{d}{dx} \ln ax = \frac{1}{x} \]

Integrals:

\[ \int a x^n \, dx = \frac{a}{n+1} x^{n+1} \]
\[ \int \frac{dx}{x} = \ln x \]
\[ \int \sin ax \, dx = -\frac{1}{a} \cos ax \]
\[ \int \cos ax \, dx = \frac{1}{a} \sin ax \]
\[ \int e^{ax} \, dx = \frac{1}{a} e^{ax} \]
\[ \int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} \]
\[ \int \frac{dx}{\sqrt{x^2 + a^2}} = \ln \left( \sqrt{x^2 + a^2} + x \right) \]
\[ \int \frac{dx}{x^2 + a^2} = \frac{1}{a} \arctan \frac{x}{a} \]
\[ \int \frac{x \, dx}{(x^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}} \]

Constants:

\[ \epsilon_0 = 8.8542 \times 10^{-12} \text{ C}^2/(\text{N m}^2) \]
\[ \mu_0 = 4\pi \times 10^{-7} \text{ Wb}/(\text{A m}) \]
\[ c = 2.9979 \times 10^8 \text{ m/s} \]

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Electromagnetic waves:
Maxwell’s equations predict the existence of electromagnetic waves that propagate in vacuum with the electric and magnetic fields perpendicular and with ratio:

\[ E = cB \]

The waves travel with velocity \( c \) where

\[ c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \]

Energy in Electromagnetic waves:
The energy flow rate (power per unit area) of an electromagnetic wave is given by the Poynting vector \( \vec{S} \)

\[ \vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \]

The magnitude of the time-averaged value of \( \vec{S} \) is called the intensity of the wave

\[ I = \frac{1}{2} \frac{E_{\text{max}} B_{\text{max}}}{\mu_0} = \frac{E_{\text{max}}^2}{2 \mu_0 c} = \frac{1}{2} \epsilon_0 c E_{\text{max}}^2 \]

Speed of light in materials
When light propagates through a material, its speed is lower than the speed in free space space by a factor called the index of refraction

\[ v = \frac{c}{n} \]

Reflection and refraction
At a smooth interface, the incident, reflected, and refracted rays and the normal to the interface all lie in a single plane. The angle of incidence and angle of reflection (measured from the normal) are equal \( \theta_i = \theta_r \) and the angle of refraction is given by Snell’s law:

\[ n_a \sin \theta_a = n_b \sin \theta_b \]

Polarization
A polarizing filter passes waves that are linearly polarized along its polarizing axis. When polarized light of intensity \( I_{\text{max}} \) is incident on a polarizing filter used as an analyzer, the intensity \( I \) of the light transmitted depends on the angle \( \phi \) between the polarization direction of the incident light and the polarizing axis of the analyzer:

\[ I = I_{\text{max}} \cos^2 \phi \]

Spherical Mirrors
Object and image distances:

\[ \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \]

where \( f = R/2 \).

Thin Lenses
Object and image distances:

\[ \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \]

where

\[ \frac{1}{f} = (n - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \]

Magnification
The lateral magnification for the systems described above is

\[ m = \frac{y'}{y} = -\frac{s'}{s} \]
Physics 208 — Formula Sheet for Exam 3
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**Forces:**
The force on a charge $q$ moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is
\[
\vec{F} = q\vec{v} \times \vec{B}
\]
and the force on a differential segment $d\vec{l}$ carrying current $I$ is
\[
d\vec{F} = I d\vec{l} \times \vec{B}
\]

**Magnetic Flux:**
Magnetic flux is defined analogously to electric flux (see formula sheet 1)
\[
\Phi_B = \int \vec{B} \cdot d\vec{A}
\]
The magnetic flux through a closed surface seems to be zero
\[
\oint \vec{B} \cdot d\vec{A} = 0
\]

**Magnetic dipoles:**
A current loop creates a magnetic dipole $\vec{\mu} = I\vec{A}$ where $I$ is the current in the loop and $\vec{A}$ is a vector normal to the plane of the loop and equal to the area of the loop. The torque on a magnetic dipole in a magnetic field is
\[
\vec{\tau} = \vec{\mu} \times \vec{B}
\]

**Biot-Savart Law:**
The magnetic field $d\vec{B}$ produced at point $P$ by a differential segment $d\vec{l}'$ carrying current $I$ is
\[
d\vec{B} = \frac{\mu_0 I d\vec{l}' \times \hat{r}}{4\pi r^2}
\]
where $\hat{r}$ points from the segment $d\vec{l}'$ to the point $P$.

**Magnetic field produced by a moving charge:**
Similarly, the magnetic field produced at a point $P$ by a moving charge is
\[
\vec{B} = \frac{\mu_0 q \vec{v} \times \hat{r}}{4\pi r^2}
\]

**Ampère’s Law:** (without displacement current)
\[
\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}
\]

**Faraday’s Law:**
The EMF produced in a closed loop depends on the change of the magnetic flux through the loop
\[
\mathcal{E} = -\frac{d\Phi_B}{dt}
\]

When an EMF is produced by a changing magnetic flux there is an induced, nonconservative, electric field $\vec{E}$ such that
\[
\oint \vec{E} \cdot d\vec{l}' = -\frac{d}{dt} \int_A \vec{B} \cdot d\vec{A}
\]

**Mutual Inductance:**
When a changing current $i_1$ in circuit 1 causes a changing magnetic flux in circuit 2, and vice-versa, the induced EMF in the circuits is
\[
\mathcal{E}_2 = -M \frac{di_1}{dt} \quad \text{and} \quad \mathcal{E}_1 = -M \frac{di_2}{dt}
\]
where $M$ is the mutual inductance of the two loops
\[
M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_1 \Phi_{B1}}{i_1}
\]
where $N_i$ is the number of loops in circuit $i$.

**Self Inductance:**
A changing current $i$ in any circuit generates a changing magnetic field that induces an EMF in the circuit:
\[
\mathcal{E} = -L \frac{di}{dt}
\]
where $L$ is the self inductance of the circuit
\[
L = \frac{N \Phi_B}{i}
\]

For example, for a solenoid of $N$ turns, length $l$, area $A$, Ampère’s law gives $B = \mu_0 (N/l)i$, so the flux is $\Phi_B = \mu_0 (N/l)iA$, and so
\[
L = \mu_0 \frac{N^2}{l} A
\]

**LR Circuits:**
When an inductor $L$ and a resistance $R$ appear in a simple circuit, exponential energizing and de-energizing time dependences are found that are analogous to those found for $RC$-circuits. The time constant $\tau$ for energizing an LR circuit is
\[
\tau = \frac{L}{R}
\]

**LC Circuits:**
When an inductor $L$ and a capacitor $C$ appear in a simple circuit, sinusoidal current oscillation is found with frequency $f$ such that
\[
2\pi f = \frac{1}{\sqrt{LC}}
\]
Capacitance:
A capacitor is any pair of conductors separated by an insulating material. When the conductors have equal and opposite charges $Q$ and the potential difference between the two conductors is $V_{ab}$, then the definition of the capacitance of the two conductors is

$$C = \frac{Q}{V_{ab}}$$

The energy stored in the electric field is

$$U = \frac{1}{2}CV^2$$

If the capacitor is made from parallel plates of area $A$ separated by a distance $d$, where the size of the plates is much greater than $d$, then the capacitance is given by

$$C = \varepsilon_0 A/d$$

Capacitors in series:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$$

Capacitors in parallel:

$$C_{eq} = C_1 + C_2 + \ldots$$

If a dielectric material is inserted, then the capacitance increases by a factor of $K$ where $K$ is the dielectric constant of the material

$$C = KC_0$$

Current:
When current flows in a conductor, we define the current as the rate at which charge passes:

$$I = \frac{dQ}{dt}$$

We define the current density as the current per unit area, and can relate it to the drift velocity of charge carriers by

$$\vec{j} = nq\vec{v}_d$$

where $n$ is the number density of charges and $q$ is the charge of one charge carrier.

Ohm’s Law and Resistance:
Ohm’s Law states that a current density $J$ in a material is proportional to the electric field $E$. The ratio $\rho = E/J$ is called the resistivity of the material. For a conductor with cylindrical cross section, with area $A$ and length $L$, the resistance $R$ of the conductor is

$$R = \frac{\rho L}{A}$$

A current $I$ flowing through the resistor $R$ produces a potential difference $V$ given by

$$V = IR$$

Resistors in series:

$$R_{eq} = R_1 + R_2 + \ldots$$

Resistors in parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$$

Power:
The power transferred to a component in a circuit by a current $I$ is

$$P = VI$$

where $V$ is the potential difference across the component.

Kirchhoff’s rules:
The algebraic sum of the currents into any junction must be zero:

$$\sum I = 0$$

The algebraic sum of the potential differences around any loop must be zero.

$$\sum V = 0$$

RC Circuits:
When a capacitor $C$ is charged by a battery with EMF given by $\mathcal{E}$ in series with a resistor $R$, the charge on the capacitor is

$$q(t) = C\mathcal{E} \left(1 - e^{-t/RC}\right)$$

where $t = 0$ is when the charging starts.

When a capacitor $C$ that is initially charged with charge $Q_0$ discharges through a resistor $R$, the charge on the capacitor is

$$q(t) = Q_0e^{-t/RC}$$

where $t = 0$ is when the discharging starts.
Physics 208 — Formula Sheet for Exam 1

Do NOT turn in these formula sheets!

Force on a charge:
An electric field \( \vec{E} \) exerts a force \( \vec{F} \) on a charge \( q \) given by:

\[
\vec{F} = q \vec{E}
\]

Coulomb’s law:
A point charge \( q \) located at the coordinate origin gives rise to an electric field \( \vec{E} \) given by

\[
\vec{E} = \frac{q}{4\pi\epsilon_0 r^2} \hat{r}
\]

where \( r \) is the distance from the origin (spherical coordinate), \( \hat{r} \) is the spherical unit vector, and \( \epsilon_0 \) is the permittivity of free space:

\[
\epsilon_0 = 8.8542 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)
\]

Superposition:
The principle of superposition of electric fields states that the electric field \( \vec{E} \) of any combination of charges is the vector sum of the fields caused by the individual charges

\[
\vec{E} = \sum_i \vec{E}_i
\]

To calculate the electric field caused by a continuous distribution of charge, divide the distribution into small elements and integrate all these elements:

\[
\vec{E} = \int d\vec{E} = \int \frac{dq}{4\pi\epsilon_0 r^2} \hat{r}
\]

Electric flux:
Electric flux is a measure of the “flow” of electric field through a surface. It is equal to the product of the area element and the perpendicular component of \( \vec{E} \) integrated over a surface:

\[
\Phi_E = \int E \cos \phi \, dA = \int \vec{E} \cdot \hat{n} \, dA = \int \vec{E} \cdot d\vec{A}
\]

where \( \phi \) is the angle from the electric field \( \vec{E} \) to the surface normal \( \hat{n} \).

Gauss’ Law:
Gauss’ law states that the total electric flux through any closed surface is determined by the charge enclosed by that surface:

\[
\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}
\]

Electric conductors:
The electric field inside a conductor is zero. All excess charge on a conductor resides on the surface of that conductor.

Electric Potential:
The electric potential is defined as the potential energy per unit charge. If the electric potential at some point is \( V \) then the electric potential energy at that point is \( U = qV \). The electric potential function \( V(\vec{r}) \) is given by the line integral:

\[
V(\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{E} \cdot d\vec{l} + V(\vec{r}_0)
\]

Beware of the minus sign. This gives the potential produced by a point charge \( q \):

\[
V = \frac{q}{4\pi\epsilon_0 r}
\]

for a collection of charges \( q_i \)

\[
V = \sum_i \frac{q_i}{4\pi\epsilon_0 r_i}
\]

and for a continuous distribution of charge

\[
V = \int \frac{dq}{4\pi\epsilon_0 r}
\]

where in each of these cases, the potential is taken to be zero infinitely far from the charges.

Field from potential:
If the electric potential function is known, the vector electric field can be derived from it:

\[
E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}
\]

or in vector form:

\[
\vec{E} = -\left( \frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{k} \right)
\]

Beware of the minus sign.