You are graded on your work, with partial credit. See the last pages of the exam for formula sheets. 
Please be clear and well-organized, so that we can easily follow each step of your work.

1. A solid conducting sphere carrying charge $Q$ has radius $a$. It is inside a concentric hollow conducting sphere with inner radius $b$ and outer radius $c$. The hollow sphere has no net charge. Use Gauss’s law (and what you know about conductors) to derive expressions for the electric field magnitude $E$ in each of the following regions.

You should include sketches which show the appropriate charges and appropriate Gaussian surfaces.

(a) (5) $r < a$:

(b) (5) $a < r < b$:
(c) (5) $b < r < c$:

(d) (5) $r > c$:

(e) (5) Sketch a graph of $E$ versus $r$ from $r = 0$ to $r = 2c$. 
2. (25) A small sphere with mass 5.0 grams hangs by a thread between two parallel vertical plates 10.0 cm apart. The plates are conducting and have uniform surface charge densities $+\sigma$ and $-\sigma$. The charge on the sphere is $q = 2.0 \times 10^{-5}$ coulomb. The thread makes an angle of $30^\circ$ with respect to a vertical line.

(a) (5) Calculate the vertical component $T_y$ of the tension in the thread.

(b) (5) Calculate the horizontal component $T_x$ of the tension in the thread.

(c) (5) Calculate the electric field between the plates.

(d) (5) Calculate the potential difference between the plates.

(e) (5) Calculate the surface charge density $\sigma$. 


3. (25) An electron is projected with an initial speed \( v_0 = 3.0 \times 10^6 \) m/s into a uniform electric field \( E \) between two parallel plates. It enters at a point midway between the plates, which are separated by 2.0 cm, and its initial velocity is parallel to the plates. The plates extend for 5.0 cm. If the electron just misses the upper plate as it emerges from the field, find the magnitude of the electric field.
4. An electric charge $Q$ is distributed uniformly on a thin rod which extends along the $y$ axis from $y = 0$ to $y = a$.

(a) (10) Calculate the potential $V(x,0,0)$ at an arbitrary point along the $x$ axis.

(This notation indicates that only $x$ is allowed to vary, with the two other coordinates both being kept equal to zero, since the points where the potential is evaluated are restricted to the $x$ axis.)

\[
\int \frac{dy}{\left(x^2 + y^2\right)^{1/2}} = \ln\left(y + \left(x^2 + y^2\right)^{1/2}\right) \quad \int \frac{dy}{x^2 + y^2} = \frac{1}{x} \arctan\left(\frac{y}{x}\right) \quad \int \frac{dy}{\left(x^2 + y^2\right)^{3/2}} = \frac{1}{x^2} \frac{y}{\left(x^2 + y^2\right)^{1/2}}
\]
(b) (10) Using your expression for the potential $V(x)$ from Part (a), calculate $E_x$, the $x$ component of the electric field (for points along the $x$ axis). [As a check on your answer, you should get $kQ/x^2$ in the limit of large $x$, after doing the algebra carefully.]

(c) (5) Can you use the result of Part (a) to calculate the other components $E_y$ and $E_z$? In one or two sentences, explain (clearly and with precision).
5. (5) For extra credit: A materials scientist decides to model a material with ionic bonding as a system of negative and positive point charges which are held together by the mutual attraction of these charges through purely electrostatic forces.

(This might be a construction material, an electronic material, or even a high-temperature superconductor.)

Bearing in mind Einstein’s statement, “A theory should be as simple as possible, but no simpler”, discuss this model in a sentence or two.

You should give a **clear specific reason** why this model will or will not work.