Homework 8

P 2.1 – counts as 2 problems

M 10–4 see below
[\( \Lambda \) is the thermal de Broglie wavelength, \( p \) is the pressure.]

M 10–27 see next page
[10–26 is only for background.]

M 10–29 see next page
[Interpretation for us: Get \( E \) from our grand partition function for the radiation field, then use this and energy flux = \((1/4)(E/V)c\) to get the Stefan Boltzmann law, evaluating \( \sigma_B \).]

10–4. Derive Eqs. (10–11) and (10–12) from Eqs. (10–9) and (10–10).

\[
N = 2\pi \left( \frac{2m}{\hbar^2} \right)^{3/2} V \int_0^\infty \frac{\lambda \epsilon^{1/2} e^{-\beta \epsilon} d\epsilon}{1 + \lambda e^{-\beta \epsilon}} \quad (10–9)
\]

\[
pV = 2\pi kT \left( \frac{2m}{\hbar^2} \right)^{3/2} V \int_0^\infty \epsilon^{1/2} \ln(1 + \lambda e^{-\beta \epsilon}) d\epsilon \quad (10–10)
\]

\[
\rho = \frac{1}{\Lambda^3} \sum_{l=1}^{\infty} \frac{(-1)^{l+1} \lambda^l}{l^{3/2}} \quad (10–11)
\]

\[
\frac{p}{kT} = \frac{1}{\Lambda^3} \sum_{l=1}^{\infty} \frac{(-1)^{l+1} \lambda^l}{l^{5/2}} \quad (10–12)
\]

[next page]
10–26. Show that the Planck blackbody distribution can be written in terms of wavelengths $\lambda$ rather than frequency:

$$\rho(\lambda, T) d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{e^{h\omega/\lambda kT} - 1}$$

where $\rho(\lambda, T) d\lambda$ is the amount of energy between wavelength $\lambda$ and $\lambda + d\lambda$.

10–27. If $\omega_{\text{max}}$ is the frequency at which $\rho(\omega, T)$ is a maximum, illustrate by maximizing $\ln \rho(\omega, T)$ that $\omega_{\text{max}}$ is given by

$$\frac{\hbar \omega_{\text{max}}}{kT} = 3(1 - e^{-h\omega_{\text{max}}/kT})$$

and so

$$\frac{\hbar \omega_{\text{max}}}{kT} = 2.82$$

Similarly show that

$$\lambda_{\text{max}} T = 0.290 \text{ cm-deg}$$

10–29. In Problem 7–24, it was shown that the number of molecules striking a surface per unit area per unit time is $\rho\rho/4$. By a similar approach, show that the total energy flux radiated by a blackbody is

$$e(T) = \frac{c E}{4V} = \sigma T^4$$

where $\sigma = 2\pi^2 k^4/15h^3c^3$. This result is known as the Stefan-Boltzmann law, and $\sigma$ is the Stefan-Boltzmann constant. Verify that $\sigma$, a universal constant, equals $5.669 \times 10^{-5}$ erg/cm$^2$-deg$^4$-sec.