7-32. Consider a two-dimensional harmonic oscillator with Hamiltonian

\[ H = \frac{1}{2m} (p_x^2 + p_y^2) + \frac{k}{2} (x^2 + y^2) \]

According to the principle of equipartition of energy, the average energy will be \(2kT\). Now transform this Hamiltonian to plane polar coordinates to get

\[ H = \frac{1}{2m} \left( p_r^2 + \frac{p_\theta^2}{r^2} \right) + \frac{k}{2} r^2 \]

What would you predict for the average energy now? Show by direct integration in plane polar coordinates that \( \bar{e} = 2kT \). Is anything wrong here? Why not?

8-15. The classical rotational kinetic energy of a symmetric top molecule is

\[ K = \frac{p_\theta^2}{2I_A} + \frac{(p_\phi - p_\psi \cos \theta)^2}{2I_A \sin^2 \theta} + \frac{p_\psi^2}{2I_C} \]

where \(I_A, I_A, \) and \(I_C\) are the principal moments of inertia, and \(\theta, \phi, \) and \(\psi\) are the three Euler angles. Derive the classical limit of the rotational partition function for a symmetric top molecule. Hint: Recall that the Euler angles have the ranges:

\[ 0 \leq \theta \leq \pi \]
\[ 0 \leq \phi \leq 2\pi \]
\[ 0 \leq \psi \leq 2\pi \]

8-16. The classical Hamiltonian for an asymmetric top molecule with principal moments of inertia \(I_A, I_B, \) and \(I_C\) is given by

\[ H = \frac{1}{2I_A \sin^2 \theta} \left\{ (p_\phi - p_\psi \cos \theta) \cos \psi - p_\theta \sin \theta \sin \psi \right\}^2 \]

\[ + \frac{1}{2I_B \sin^2 \theta} \left\{ (p_\phi - p_\psi \cos \theta) \sin \psi + p_\theta \sin \theta \cos \psi \right\}^2 + \frac{1}{2I_C} p_\phi^2 \]

Derive the classical limit of the rotational partition function for an asymmetric top molecule. Hint: It may help to rearrange the Hamiltonian and integrate over \(p_\theta, p_\phi, p_\psi\) in that order.