HW 12.1. A small perturbation $\lambda x^4$ is added to the Hamiltonian for the harmonic oscillator. (This represents the anharmonic effects in a real system.)

(a) Calculate the first-order correction to the energy eigenvalues, $\Delta E_n$.

Give your answer in terms of $n$, $\lambda$, $\hbar$, the mass $m$, and the force constant $k$.

HW 12.2. For the same problem as above, calculate the leading correction to the ground-state wavefunction.

This involves calculating the matrix element $\langle m|\lambda x^4|0\rangle$ for the lowest intermediate state $m$ which gives a nonvanishing correction in lowest-order perturbation theory.

Give your answer as a coefficient (involving the same parameters as in problem 1 above) times the unperturbed wavefunction $\psi_m(x)$, which is mixed into $\psi_0(x)$ by the perturbation (with $m$ specified, of course). You do not need to normalize the corrected wavefunction to one.

HW 12.3. A hydrogen atom is placed in a uniform static electric field $\varepsilon$ that points along the z axis. The (relatively small) perturbation in the Hamiltonian is $-e\varepsilon z$, where $e$ is the fundamental charge. It removes the degeneracy of some of the states, and this phenomenon is called the Stark effect.

Using degenerate perturbation theory, calculate the lowest-order shifts in energy for the four $n = 2$ states in hydrogen. Give the answer in terms of $e$, $\varepsilon$, and the Bohr radius $a_0$.

You may use the fact that $\langle 2, \ell = 0, m = 0|z|2, \ell = 1, m = 0\rangle = -3a_0$. 