The London-Anderson-Englert-Brout-Higgs-Guralnik-Hagen-Kibble mechanism and Higgs boson reveal the unity and future excitement of physics

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The particle recently discovered by the CMS and ATLAS collaborations at CERN is almost certainly a Higgs boson, fulfilling a quest that can be traced back to three seminal high-energy papers of 1964, but which is intimately connected to ideas in other areas of physics that go back much further. One might oversimplify the history of the features which (i) give mass to the W and Z particles that mediate the weak nuclear interaction, (ii) effectively break gauge invariance, (iii) eliminate physically unacceptable Nambu–Goldstone bosons, and (iv) give mass to fermions (like the electron) by collectively calling them the London–Anderson–Englert–Brout–Higgs–Guralnik–Hagen–Kibble–Weinberg mechanism. More important are the implications for the future: a Higgs boson appears to point toward supersymmetry, since new physics is required to protect its mass from enormous quantum corrections, while the discovery of neutrino masses seems to point toward grand unification of the non-gravitational forces.

Keywords: gauge invariance; symmetry breaking; superconductivity; London equation; Higgs boson

1. Introduction

In 1935, Fritz and Heinz London [1] effectively gave mass to the photon in a superconductor, and thereby provided a macroscopic explanation of the Meissner effect—the expulsion of a magnetic field from a superconductor. From a modern perspective, this mechanism for giving mass to a vector boson can be interpreted as implying an effective breaking of gauge invariance. In 1963, following closely related treatments by himself and others (see [2] and references therein), Philip Anderson [3] pointed out another aspect important for the construction of models in particle physics: A would-be zero-mass Nambu–Goldstone boson in a superconductor is effectively eaten by the photon to become a finite-mass longitudinal mode, which appears as a plasmon in a nonrelativistic treatment. The plasma frequency,

\[ \omega_p = \sqrt{\frac{4\pi n_e e^2}{m_e}}, \]

(1)

can be interpreted as the long-wavelength limit of the frequency of a longitudinal mode which has the same mass as the transverse modes described by Equations (7) and (9) below. In 1964, realistic models with Lorentz invariance and nonabelian gauge fields were formulated by Englert and Brout [4], Higgs [5,6], and Guralnik, Hagen, and Kibble [7]. The prediction of an observable boson was made by Higgs [6,8] and emphasized by Ellis et al. [9] and others. Finally, Weinberg [10] recognized that a Yukawa interaction with the Higgs field would give masses to fermions like the electron in the fully developed electroweak theory [10–12]. So one might collectively call the set of all four essential features the London–Anderson–Englert–Brout–Higgs–Guralnik–Hagen–Kibble–Weinberg (LAE-BHGHKW) mechanism for giving masses to fundamental particles. Of course, this list leaves out the critical contributions of many others in the rich history, which has been summarized an enormous number of times in reviews and books, but with the earliest origins outside particle physics usually omitted or de-emphasized.

2. Photon mass and breaking of gauge invariance in a superconductor

Weinberg has stated numerous times that “A superconductor is simply a material in which electromagnetic gauge invariance is spontaneously broken” and has given plausibility arguments why the principal properties of a superconductor should follow from the breaking of gauge invariance [13]. From a modern perspective, this idea originates with the 1935 paper of the London brothers [1], who postulated that

\[ \nabla \times j = -\frac{n_e e^2}{mc} B \]

(2)
so that a magnetic field $\mathbf{B}$ induces diamagnetic currents $\mathbf{j}_s$. This equation follows if one assumes that [14]

$$\mathbf{j}_s = -\frac{n_e e^2}{mc} \mathbf{A}, \quad (3)$$

where $\mathbf{A}$ is the vector potential. And this last equation is obtained if we assume (in modern nomenclature) an order parameter $\psi_s$ which does not break translational invariance, in the sense that it is uniform in space (just as the Higgs vacuum expectation value $\langle \phi_{H} \rangle$ is assumed not to break Lorentz/Poincaré invariance): The electric current density is given by

$$\mathbf{j}_s = n_s q \mathbf{v}_s = n_s \frac{q}{m} \mathbf{P}_s = n_s \frac{q}{m} \left( \mathbf{p}_s - \frac{q}{c} \mathbf{A} \right), \quad (4)$$

classically, where $\mathbf{P}_s$ is the mechanical momentum and $\mathbf{p}_s$ is the canonical momentum, or

$$\mathbf{j}_s = \text{Re} \left[ \frac{q}{m} \psi_s^* \left( -i \hbar \nabla - \frac{q}{c} \mathbf{A} \right) \psi_s \right] = -n_s \frac{q^2}{mc} \mathbf{A}, \quad (5)$$

quantum mechanically, since $\nabla \psi_s = 0$. After the Ginzburg-Landau and BCS theories, and subsequent experimental discoveries, the interpretation is $q = -2e$, with $m$ an effective mass, and with the condensate of Cooper pairs corresponding to the Higgs condensate.

Strictly speaking, Equation (3) holds only in a particular gauge (the London gauge), and in an exact treatment the fundamental requirement of gauge invariance still holds both in a superconductor and in high energy physics [2,15] if the ground state or vacuum is included. However, there is an effective breaking of gauge invariance – i.e. a breaking of gauge invariance if only the excitations above the ground state or particles above the vacuum are included – which reveals itself in various ways. First, the argument in the paragraph above shows that Equation (3) follows if the order parameter is invariant under translations. That is, this kind of translational invariance in the ground state implies an equation which is manifestly not invariant under a gauge transformation $\mathbf{A} \rightarrow \mathbf{A}' = \mathbf{A} + \nabla \Lambda$. (In the present paper we ignore the rich variety of phenomena in condensed matter physics which involve ground states that are not translationally invariant or which are otherwise more complex than the simplest superconductors.) Furthermore, either Equation (2) or (3) effectively implies a mass for the photon according to the following argument: The two (static) Maxwell equations

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{j}_s, \quad \nabla \cdot \mathbf{B} = 0, \quad (6)$$

together with Equation (2) and $\nabla \times (\nabla \times \mathbf{B}) = \nabla (\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B}$, imply that

$$\nabla^2 \mathbf{B} = m_{ph}^2 \mathbf{B}, \quad m_{ph} = \frac{1}{\hbar_L}, \quad \hbar_L \equiv \sqrt{\frac{mc^2}{4\pi n_e e^2}}. \quad (7)$$

Here, $m_{ph}$ is the scaled mass, related to the mass $M_{ph}$ in standard units by $m_{ph} = M_{ph} c / \hbar$. In order to obtain Equation (7) directly we must replace the original gauge-invariant Maxwell equation (with no external current or time dependence), $\nabla \times \mathbf{B} = 0$, by

$$\nabla \times \mathbf{B} + m_{ph}^2 \mathbf{A} = 0, \quad (8)$$

which is again manifestly not gauge-invariant. Since $\mathbf{B} = \nabla \times \mathbf{A}$, this can also be written $\nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} + m_{ph}^2 \mathbf{A} = 0$. But subjecting this to $\nabla \cdot \mathbf{A}$ gives a cancellation of the first two terms, so that $\nabla \cdot \mathbf{A} = 0$ and $-\nabla^2 \mathbf{A} + m_{ph}^2 \mathbf{A} = 0$. This is the zero frequency limit of the wave equation for a massive photon

$$\frac{1}{c^2} \frac{\partial^2}{\partial t^2} \mathbf{A} - \nabla^2 \mathbf{A} + m_{ph}^2 \mathbf{A} = 0. \quad (9)$$

At zero frequency, of course, at least one wavevector component must be imaginary, and the Meissner effect follows, as in the last paragraph of this section.

The action which leads to Equation (9) also lacks gauge invariance because of the mass term. The London theory thus already contains two of the four essential features of the LAEBHGW mechanism, with (an effective) spontaneous breaking of gauge symmetry and a mass for the gauge boson, which is in this case the photon.

In the electroweak theory, the vacuum, and thus the vacuum expectation value $\langle \phi_H \rangle$ of the Higgs field, are typically required to have Lorentz/Poincaré invariance. This requirement then leads to the result that gauge invariance is broken. Similarly, when the order parameter $\psi_s$ in a superconductor is required to have translational invariance, Equation (5) shows that gauge invariance is broken. The requirement that $\psi_s$ be invariant under translations is less compelling for a superconductor, because $|\psi_s|^2$ rather than $\psi_s$ appears in the photon mass, whereas $\langle \phi_H \rangle$ itself appears in fermion masses according to Equation (28) below. However, $\psi_s$ can be interpreted as the expectation value for an electron-pair field, and it is natural to require that it also be translationally invariant in the ground state. (It should be emphasized that all the reasoning here is for a ground state of either the superconductor or the universe.) One might adopt the position that gauge invariance is broken if a more fundamental requirement, translational invariance of the vacuum or ground state, is to be preserved.
As a final connection with the electroweak theory, suppose that a current $j$ of electron quasiparticles (and quasiholes) is added to the modified Maxwell equation, Equation (8):

$$\nabla \times \mathbf{B} + m^2 \rho \mathbf{A} = \frac{4\pi}{c} j.$$  \hspace{1cm} (10)

Applying $\nabla \cdot$ gives $\nabla \cdot j \propto \nabla \cdot A$, and then by Equation (3) $\nabla \cdot (j + j) = 0$, rather than $\nabla \cdot j = 0$, so the quasiparticle current is not conserved. Instead, the condensate acts essentially as a reservoir of electron Cooper pairs. This is, of course, another result of the effective breaking of gauge invariance: a conservation law required by symmetry, according to Noether’s theorem, no longer holds when the symmetry is broken. That is, if the ‘vacuum’ is included, charge is conserved, but charge is not conserved for excitations above the ‘vacuum’, which are described by $j$. This basic effect is displayed in Andreev reflection, where a negatively-charged electron is reflected as a positively-charged hole. In the same way, the initial conservation laws for the (1) weak hypercharge $Y$ and the SU(2) weak isospin no longer hold after the Higgs condensate forms, and all that is left is conservation of the electric charge $Q = T^3 + Y$.

For a geometry with a planar boundary at $x = 0$, and variation only in the $x$ direction, the solution of Equation (7) is $B = B(0)e^{-x/\lambda}$, so the magnetic field falls to zero inside the superconductor with a London penetration depth $\lambda$. It is a true demonstration of the unity of physics that this Meissner effect in a superconducting metal and the short range of the weak nuclear force in the universe have the same origin: in each case the vector bosons (photons or W and Z bosons) grow masses because they are coupled to a field which forms a condensate at low temperature, as the metal is cooled in the laboratory or the universe expands and cools after the Big Bang.

3. Origin of the masses of fundamental particles

The mass of an atom or human body arises about 99% from the energy of quarks and gluons moving relativistically inside protons and neutrons, in accordance with $E = mc^2$. The mass of an electron, on the other hand, arises from its Yukawa coupling to the Higgs field. The radius of an electron’s orbit in the ground state of a hydrogen atom is

$$r = \frac{\hbar^2}{m_e e^2}.$$ \hspace{1cm} (11)

and similar results hold for other atoms. So if the mass $m_e$ of an electron were zero there would be no atoms, and the formation of ordinary matter would be impossible without the Higgs condensate.

In the standard model of particle physics [16,17] (see [18] for reviews and references to the original papers for the well-known topics beyond this point), scalar bosons are coupled to the gauge bosons through the covariant derivative $D_{\mu} = \partial_{\mu} - igA_{\mu}^i$, in the action

$$S = \int d^4x \phi^*_h(x) D^\mu D_{\mu} \phi_h(x).$$ \hspace{1cm} (12)

There is thus a term proportional to $\phi^*_h(x) A_{\mu}^\alpha A_{\mu}^\beta \epsilon^{\alpha\beta\mu\nu} \phi_h(x)$ which has the potential to become a mass term with the form $m^2 A_{\mu}^\alpha A_{\mu}^\beta$ if (i) the scalar boson field $\phi_h$ undergoes condensation, acquiring a nonzero vacuum expectation value, and (ii) the generators $t^i$ behave properly. This happens in the electroweak theory because the remaining action for the Higgs field has the Ginzburg–Landau form $-\mu^2 \phi^*_h \phi_h + \frac{1}{2} \lambda (\phi^*_h \phi_h)^2$, and $t^i = \sigma^i/2$ in the non-abelian part of $D_{\mu}$, where the $\sigma^i$ are the Pauli matrices.

More precisely, in the electroweak theory the covariant derivative is

$$D_{\mu} = \partial_{\mu} - igA_{\mu}^i T^i - igB_{\mu} Y,$$ \hspace{1cm} (13)

where $T^i$ and $Y$ are, respectively, the operators for the SU(2) weak isospin and U(1) weak hypercharge. In the representation with weak isospin $\frac{1}{2}$, to which the Higgs field belongs, the generators are $T^i = \frac{1}{2}\sigma^i$, with $i = 1, 2, 3$, and with the same notation used for operators and their matrix representations. The Higgs field also has weak hypercharge $\frac{1}{2}$, so

$$D_{\mu} \phi_H = \left( \partial_{\mu} - igA_{\mu}^i \frac{\sigma^i}{2} - igB_{\mu} Y \right) \phi_H.$$ \hspace{1cm} (14)

with

$$\langle \phi_H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu \end{pmatrix}$$ \hspace{1cm} (15)

after symmetry-breaking, where the treatment here and below is restricted to the unitarity gauge (just as the treatment of a superconductor was restricted to the London gauge). Algebra then gives a term

$$\frac{1}{2} \nu^2 \left[ g^2 (A_{\mu}^1)^2 + g^2 (A_{\mu}^2)^2 + (-gA_{\mu}^1 + g'B_{\mu})^2 \right]$$ \hspace{1cm} (16)

in the action. This expression can be rewritten in terms of mass and charge eigenstates, which are linear combinations of the original SU(2) and U(1) fields, with the first three mediating the weak nuclear interaction and the fourth being the photon.
\[ W^\pm_\mu = \frac{1}{\sqrt{2}} (A^\mu_\mu + iA^\mu_\mu) \] with mass \( m_W = g \frac{\sqrt{2}}{2} \). \( \text{Eq. (17)} \)

\[ Z^0_\mu = \frac{1}{\sqrt{g^2 + g'^2}} (gA^\mu_\mu - g'B_\mu) \] with mass \( m_Z = \sqrt{g^2 + g'^2} \frac{\sqrt{2}}{2} \). \( \text{Eq. (18)} \)

\[ A_\mu = \frac{1}{\sqrt{g^2 + g'^2}} (g'A^\mu_\mu + gB_\mu) \] with mass \( m_A = 0 \). \( \text{Eq. (19)} \)

The electric charge operator is defined by

\[ Q = T^3 + Y. \] \( \text{Eq. (20)} \)

With

\[ T^\pm = T^1 \pm iT^2 = \frac{1}{2} (\sigma^1 \pm i\sigma^2), \] \( \text{Eq. (21)} \)

algebra then gives

\[ D_\mu = \partial_\mu - i\frac{g}{\sqrt{2}} (W^+_\mu T^+ + W^-_\mu T^-) \]

\[ -i\frac{g}{\cos \theta_w} Z_\mu (T^3 - \sin^2 \theta_w Q) - ieA_\mu Q, \] \( \text{Eq. (22)} \)

where the fundamental electric charge \( e \) and weak mixing angle \( \theta_w \) are defined by

\[ e = g \sin \theta_w, \]

\[ \sin \theta_w = \frac{g'}{\sqrt{g^2 + g'^2}}, \cos \theta_w = \frac{g}{\sqrt{g^2 + g'^2}}, \] \( \text{Eq. (23)} \)

so that

\[ \begin{pmatrix} Z^0 \\ A \end{pmatrix} = \begin{pmatrix} \cos \theta_w & -\sin \theta_w \\ \sin \theta_w & \cos \theta_w \end{pmatrix} \begin{pmatrix} A^1 \\ B \end{pmatrix}. \] \( \text{Eq. (24)} \)

A critical feature is that only the 2-component left-handed parts of the fermion fields experience the \( SU(2) \) weak interaction. In the Weyl representation the Dirac equation is

\[ \begin{pmatrix} -m_f & i\sigma \cdot \partial \\ i\sigma \cdot \partial & -m_f \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = 0 \] \( \text{Eq. (25)} \)

with \( \sigma^a = (1, \sigma) \) and \( \bar{\sigma} = (1, -\sigma) \) in a standard notational convention, so the fermion mass \( m_f \) couples left- and right-handed fields (as is required by Lorentz invariance). The two-component right-handed fermion fields are \( SU(2) \) singlets \( \epsilon_{\nu_R, u_R, d_R} \) (with weak isospin = 0) and the left-handed fields are placed into doublets:

\[ E_L = \begin{pmatrix} v_e \\ e^- \end{pmatrix} \] and \( Q_L = \begin{pmatrix} u \\ d \end{pmatrix} \)

with \( t^3 = \pm \frac{1}{2} \) and \( y = -\frac{1}{2} \) or \( y = +\frac{1}{6} \). \( \text{Eq. (26)} \)

where \( t^3 \) and \( y \) are respectively the eigenvalues of \( T^3 \) and \( Y \). Here the upper (or lower) sign corresponds to the upper (or lower) component of each field, and both components of a given doublet have the same weak hypercharge. Recall that \( Q = T^3 + Y \), or \( q = t^3 + y \), so we get the correct charges \( q \) for neutrino \( v_e \), electron \( e^- \), up quark, and down quark. The Higgs field is also a doublet:

\[ \phi_H = \frac{1}{\sqrt{2}} \begin{pmatrix} -i(\phi_1^1 - i\phi_1^2) \\ v \end{pmatrix} \]

\[ \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \] in the ground state \( \text{Eq. (27)} \)

where \( v \) represents the massive Higgs boson. The 3 \( \phi_i \) are would-be Nambu–Goldstone bosons, which are eaten by the \( W^+, W^-, \) and \( Z^0 \) vector bosons when they become massive and thus acquire longitudinal polarizations, so that there is no physical Nambu–Goldstone boson.

Notice that a fermion mass term, \( -m_f \overline{\psi}_L \psi_R - m_f \overline{\psi}_R \psi_L \), in the action again violates gauge invariance, since \( \psi_L \) behaves differently after \( \psi_R \) under a gauge transformation. The natural way to achieve fermion masses \( \text{[10]} \) is to postulate Yukawa couplings with the form

\[ -\lambda_\nu \overline{E}_L \cdot \phi_H e_R + \text{h.c.} \] \( \text{Eq. (28)} \)

(where h.c. means Hermitian conjugate), which after symmetry breaking becomes

\[ -m_e \overline{e}_L e_R + \text{h.c.}, \quad m_e = \frac{1}{2} \lambda_\nu v. \] \( \text{Eq. (29)} \)

So now the electron and other fermions can have mass, and the weak nuclear force is very short range (mediated by force-carrying particles which have very large masses), all because the Higgs field condensed, acquiring a large vacuum expectation value as the universe cooled after the Big Bang.

4. Future physics related to the Higgs in various ways: supersymmetry, grand unification, dark energy, quantum gravity

The discovery of a scalar boson immediately points to physics beyond the standard model, since otherwise radiative corrections should push the mass of this particle up to a ridiculously large value \( \text{[18]} \). The most natural
candidate for such new physics is supersymmetry (susy), for which there is already indirect experimental evidence, in the sense that the coupling constants of the three non-gravitational forces are found to converge to a common value (as they are run up to high energy in a grand unified theory) only if the calculation includes susy. In addition, susy predicts a neutralino which is an extremely natural candidate for dark matter. So, instead of addition, susy predicts a neutralino which is an extreme

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In summary, the discovery of the Higgs boson is a strong reminder of both the essential unity of physics and the twenty-first century mysteries that should ultimately lead to a deeper understanding of nature.

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and (ii) why is there not a cosmological constant due to (negative) vacuum energy density, which should show up in the vacuum energy which is roughly 50 or even 120 orders of magnitude larger than the observed dark energy? This second mystery was taken seriously after it was recognized that the Higgs condensate has an enormous (negative) vacuum energy density, which should show up gravitationally according to conventional physics. So the cosmological constant and dark energy problems are yet again associated with the Higgs phenomenon.

In summary, the discovery of the Higgs boson is a strong reminder of both the essential unity of physics and the twenty-first century mysteries that should ultimately lead to a deeper understanding of nature.
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