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Maxwell's Equations

Total change $Q_{\text{total}}$ (Gauss's Law).

\[ Q_{\text{total}} = \int_{\text{volume}} \rho(x,y,z) \, dx \, dy \, dz = \int_{\text{surface}} \varepsilon_0 E \cdot \hat{n} \, ds \]

- $\rho(x,y,z)$ charge density in space.
- $\varepsilon_0 E$ electric field.
- $\hat{n}$ normal unit vector of surface.

Contributions from surfaces perpendicular to $x$-axis

\[ \int_{\Omega} E \cdot \hat{n} \, ds + \int_{\Omega_2} E \cdot \hat{n} \, ds \]

Since $\hat{n}$ is parallel to $x$-axis then $E \cdot \hat{n}$ for these integrals is just $E_x$ and $ds = dy \, dz$.

\[ \int E_x (x+\Delta x, y, z) \, dy \, dz + \int -E_x (x, y, z) \, dy \, dz \]

\[ E_x(x+\Delta x, y, z) \Delta y \, \Delta z - E_x(x, y, z) \Delta y \, \Delta z \]

\[ \left[ E_x(x+\Delta x, y, z) - E_x(x, y, z) \right] \Delta y \, \Delta z \]

\[ \frac{\partial E_x}{\partial x} \Delta x \, dy \, \Delta z. \]
Repeating for all surfaces,
\[ \int E \cdot \hat{n} \, ds = \left( \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \right) \Delta x \Delta y \Delta z \]
\[ = (\nabla \cdot E) \Delta x \Delta y \Delta z \]

where \( \nabla = \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z} \) and so since
\[ \int E \cdot \hat{n} \, ds = \int \rho/\varepsilon_0 \, dx \, dy \, dz \sim \rho/\varepsilon_0 \Delta x \Delta y \Delta z \]
we have that
\[ \nabla \cdot E = \rho/\varepsilon_0 \]

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Magnetic Fields (Faraday's Law).

\[ \frac{d \Phi}{dt} = \frac{d}{dt} \int B \cdot \hat{n} \, ds = \oint E \cdot d\ell \]

\[ \frac{d \Phi}{dt} = \frac{d}{dt} \int B \cdot \hat{n} \, ds = \oint E \cdot d\ell \]

\[ \frac{d \Phi}{dt} = \frac{d}{dt} \]
\[ \oint E \cdot d\ell = \int_{(x,y)}^{x+\delta x,y} E_x \, dx + \int_{x+\delta x,y}^{x+\delta x,y+\delta y} E_y \, dy + \int_{x+\delta x,y+\delta y}^{x,y+\delta y} E_x \, dx + \int_{x,y+\delta y}^{x,y} E_y \, dy \]

\[ = \int_{x}^{x+\delta x} \left[ E_x(x', y) - E_x(x', y + \delta y) \right] \, dx + \int_{y}^{y + \delta y} \left[ E_y(x + \delta x, y') - E_y(x, y') \right] \, dy \]

\[ = \int_{x}^{x+\delta x} dx \left[ - \frac{\partial E_x}{\partial y} \delta y \right] + \int_{y}^{y + \delta y} dy \left[ \frac{\partial E_y}{\partial x} \delta x \right] \]

\[ \oint E \cdot d\ell = - \frac{\partial E_x}{\partial y} \delta x \delta y + \frac{\partial E_y}{\partial x} \delta x \delta y. \]

\[ \oint E \cdot d\ell = - \frac{dB_z}{dt} \oint B \cdot \hat{n} \, ds = - \frac{dB_z}{dt} (B_z \delta x \delta y) \]

\[ - \frac{dB_z}{dt} = \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \]

For loop with orumbahoe in \( x \) and \( y \) directions

\[ - \frac{dB_x}{dt} = \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \]

\[ - \frac{dB_y}{dt} = \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \]
The equation can be summarized as
\[ \frac{\partial B}{\partial t} = \nabla \times E \] (Curl of \( E \))

\[ \nabla \times E = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \hat{x} \left[ \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right] + \hat{y} \left[ \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right] + \hat{z} \left[ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right] \]

\[ \oint B \cdot dl = \mu_0 i_{\text{enclosed}} + \mu_0 \varepsilon_0 \int \frac{\partial E}{\partial t} \cdot \hat{n} \, ds \]

Sources of magnetic field include currents and changing electric flux.

Comparing this with the equation
\[ \oint E \cdot dl = -\frac{\partial}{\partial t} \int B \cdot \hat{n} \, ds \Rightarrow \nabla \times E = -\frac{\partial B}{\partial t} \]

\[ \oint B \cdot dl = \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \int E \cdot \hat{n} \, ds + \mu_0 \int \mathbf{J}_e \cdot \hat{n} \, ds \]

\[ \nabla \times B = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} + \mu_0 \mathbf{J}_e \]

Maxwell's Eq.
\[ \nabla \cdot E = \rho/\varepsilon_0 \quad \nabla \cdot B = 0 \quad (\text{No magnetic charge}) \]

\[ \nabla \times E = -\frac{\partial B}{\partial t} \quad \nabla \times B = \mu_0 \mathbf{J}_e + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \]
Assume no charges or curvets (free space).

\[
\frac{\partial^2 B_z}{\partial t^2} = \frac{1}{\mu_0} \left[ \frac{\partial B_z}{\partial t} \right] \quad \text{but} \quad \frac{\partial B_z}{\partial t} = -\nabla \times E
\]

\[
= \frac{\partial}{\partial t} \left[ - \left( \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) \right]
\]

\[
= - \left[ \frac{\partial}{\partial x} \left( \frac{\partial E_y}{\partial t} \right) - \frac{\partial}{\partial y} \left( \frac{\partial E_x}{\partial t} \right) \right]
\]

but \( \frac{\partial E}{\partial t} = \frac{1}{\mu_0 \varepsilon_0} \nabla \times B \).

\[
\frac{\partial^2 B_x}{\partial t^2} = -\frac{1}{\mu_0 \varepsilon_0} \left[ \frac{\partial}{\partial x} \left[ \frac{\partial B_x}{\partial t} - \frac{\partial B_y}{\partial x} \right] - \frac{\partial}{\partial y} \left[ \frac{\partial B_x}{\partial y} - \frac{\partial B_z}{\partial y} \right] \right]
\]

\[
= -\frac{1}{\mu_0 \varepsilon_0} \left[ \frac{\partial^2 B_x}{\partial x \partial t} - \frac{\partial^2 B_y}{\partial x \partial y} - \frac{\partial^2 B_z}{\partial y \partial t} \right]
\]

\[
= +\frac{1}{\mu_0 \varepsilon_0} \left[ \left( \frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_y}{\partial y^2} \right) - \frac{\partial}{\partial y} \left( \frac{\partial B_x}{\partial y} + \frac{\partial B_y}{\partial y} \right) \right]
\]

But \( \nabla \cdot B = 0 \) and so \( \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} = -\frac{\partial B_z}{\partial z} \) and

\[
\frac{\partial^2 B_z}{\partial t^2} = \frac{1}{\mu_0 \varepsilon_0} \left[ \left( \frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_y}{\partial y^2} + \frac{\partial^2 B_z}{\partial z^2} \right) \right]
\]

\[
= \frac{1}{\mu_0 \varepsilon_0} \left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right] B_z
\]

\[
\nabla \cdot \nabla
\]

\[
\frac{\partial^2 B_z}{\partial t^2} = \frac{1}{\mu_0 \varepsilon_0} \nabla^2 B_z. \quad \text{or in general}
\]

\[
\frac{\partial^2 B}{\partial t^2} = \frac{1}{\mu_0 \varepsilon_0} \nabla^2 B. \quad \text{and} \quad \frac{\partial^2 E}{\partial t^2} = \frac{1}{\mu_0 \varepsilon_0} \nabla^2 E.
\]
If $B = B_x(z)$ is only a function of $z$ then

$$\frac{\partial^2 B_x}{\partial t^2} = \frac{1}{\mu_0} \nabla^2 B \Rightarrow \frac{\partial^2 B_x}{\partial t^2} = \frac{1}{\mu_0} \frac{\partial^2 B_x}{\partial z^2}$$

Then any "disturbance" in the magnetic field must travel at a velocity

$$\left( \frac{1}{\mu_0 \varepsilon_0} \right)^{\frac{1}{2}} = \left( \frac{1}{8.85 \times 10^{-12} \text{ F/m} \cdot 4\pi \times 10^{-7} \text{ W/A.m}} \right)^{\frac{1}{2}} = 2.99 \times 10^8 \text{ m/s} = c \text{ (Speed of light)}.$$
Energy Considerations

Vector Identity \[ \nabla \cdot (A \times B) = B \cdot (\nabla \times A) - A \cdot (\nabla \times B) \]

Just as we found out what the appropriate energy densities were for the string by considering the wave equation.

Starting with

\[ \varepsilon_0 \frac{\partial E}{\partial t} = \frac{1}{\mu_0} \nabla \times B \quad \text{Multiply by } E \]

\[ \varepsilon_0 E \cdot \frac{\partial E}{\partial t} = \frac{1}{\mu_0} E \cdot (\nabla \times B) \quad \text{Using the identity} \]

\[ = \frac{1}{\mu_0} \left[ B \cdot (\nabla \times E) - \nabla \cdot (E \times B) \right] \]

But \[ \nabla \times E = -\frac{\partial B}{\partial t} \]

\[ \varepsilon_0 E \cdot \frac{\partial E}{\partial t} = -\frac{1}{\mu_0} B \cdot \frac{\partial B}{\partial t} - \frac{1}{\mu_0} \nabla \cdot (E \times B) \quad \text{or} \]

\[ \varepsilon_0 E \cdot \frac{\partial E}{\partial t} + \frac{1}{\mu_0} B \cdot \frac{\partial B}{\partial t} = -\frac{1}{\mu_0} \nabla \cdot (E \times B) \]

\[ \frac{1}{2} \frac{\partial}{\partial t} \left[ \frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2} \left( \frac{1}{\mu_0} \right) B^2 \right] = -\frac{1}{\mu_0} \nabla \cdot (E \times B) \]

Integrate over some volume.

\[ \frac{1}{2} \frac{\partial}{\partial t} \int \text{dVol} \left[ \frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2} \left( \frac{1}{\mu_0} \right) B^2 \right] = -\int \frac{1}{\mu_0} \nabla \cdot (E \times B) \text{dVol}. \]

But we have already seen that \[ \int \nabla \cdot A \text{dVol} = \int A \cdot v \text{dSurf} \]

\[ \frac{1}{2} \frac{\partial}{\partial t} \int \text{dVol} \left[ \frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2} \left( \frac{1}{\mu_0} \right) B^2 \right] = -\int \frac{1}{\mu_0} (E \times B) \cdot v \text{dSurf}. \]
Consider a capacitor

\[ E = \frac{\mathbf{V}}{\varepsilon_0} \quad \text{(Gauss's Law)} \]

Energy of capacitor: \[ \frac{1}{2} C V^2 \text{ or } \frac{1}{2} \frac{Q^2}{C} \quad \text{where } C = \frac{A \varepsilon_0}{d} \]

\[ \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} \left( \frac{1}{A \varepsilon_0/d} (AV)^2 \right) = \frac{1}{2} \frac{A d}{\varepsilon_0} V^2 = \frac{1}{2} \varepsilon_0 E^2 (AV) \]

So the energy of a capacitor is just \( \frac{1}{2} \varepsilon_0 E^2 \) times the volume of the capacitor!

Consequently \( \frac{1}{2} \varepsilon_0 E^2 \) is the energy/volume associated with the electric field.

Consider an inductor

\[ \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I \text{ enclosed} \quad \Rightarrow \quad \mathbf{B} \mathbf{l} = \mu_0 \pi n l \]

\[ I = \frac{B}{\mu_0 n} \quad \text{or} \quad B = \mu_0 n I \]

The inductance \( L \) of the coil is defined in terms of the emf that is associated with the changing current i.e.

\[ \text{Emf} = -L \frac{dI}{dt} \quad \text{but for the above coil the emf for each term is just } \text{Emf}_i = -\frac{d\Phi}{dt} \quad \text{where } \Phi = AB \text{ is the flux through each loop.} \]

For the entire coil then:

\[ \text{Emf} = -nl \frac{d\Phi}{dt} = -nlA \frac{dB}{dt} \]

\[ \text{Emf} = -nLA/\mu_0 \frac{dI}{dt} \quad \text{and so } \quad L = \mu_0 n^2 l A \]
The energy of an inductor is \( \frac{1}{2} L I^2 \). Using \( L = \mu_0 n^2 l A \) and \( I = \frac{B}{\mu_0 n} \) we have that

\[
\frac{1}{2} L I^2 = \frac{1}{2} \mu_0 n^2 l A \left( \frac{B}{\mu_0 n} \right)^2 = \frac{1}{2\mu_0} B^2 (lA) \]

or

the energy inside an inductor is \( \frac{1}{2\mu_0} B^2 \) times the volume of the inductor.

Consequently \( \frac{1}{2\mu_0} B^2 \) is the energy/volume associated with the magnetic field.

Looking back at our expression then

\[
\frac{\partial}{\partial t} \int d\text{Vol} \left[ \frac{1}{2} E \cdot E + \frac{1}{2\mu_0} B^2 \right]
\]

is the time rate of change of the energy in a volume.

Therefore the term \(- \int d\text{Surf} \cdot \frac{1}{\mu_0} (E \times B) \cdot \hat{n}\) must have something to do with the rate at which energy flows through the surface.

Since \( \hat{n} \) is the outward pointing normal \( \frac{1}{\mu_0} (E \times B) \) must be the rate at which energy flows per unit area.

\[
S = \frac{1}{\mu_0} (E \times B)
\]

is called the Poynting vector. The direction indicates the direction of energy flow and its magnitude represents the amount of energy that passes through a unit area per unit time.
Radiation Pressure.

\[ \frac{mv}{\theta} \rightarrow \text{The force on the wall is given by the impulse or } \frac{dP}{dt} \text{ the time rate of change of the momentum.} \]

\[ -mv \rightarrow \text{In the collision the change in the momentum of the ball is } 2mv. \]

So for a block of electromagnetic energy the total energy within the block is just \( EAL \). Its equivalent mass (using the Einstein relation) is just:

\[ EAL = \frac{m}{c^2} \] and so if this block is completely reflected by a surface the total change in its momentum is

\[ \Delta P = 2m'v = 2 \left[ \frac{EAL}{c^2} \right] c \text{ where } v = c \text{ in this case.} \]

The force then becomes

\[ \text{Force} = \frac{\Delta P}{\Delta t} \text{ where } \Delta t \text{ is the total collision time and } \Delta t = \frac{L}{c} = \Delta t \]

\[ = 2 \left[ \frac{EAL}{c^2} \right] c = 2EA \]

\[ \frac{L}{c} \]

The pressure on the surface then is

\[ \text{Pressure} = FA = 2EA \cdot \frac{2|S|}{c} \]

Note: \( S \) is the energy flux the energy momentum relationship for light is

\[ E = cp \] and the energy is proportional to its momentum. Thus,

\[ \frac{S}{c} \] is the momentum flux!
\[ T(K) = T(°C) + 273.15 \]

\[ \frac{\Delta L}{L} = \alpha \Delta T \]

\[ \frac{\Delta V}{V} = \beta \Delta T = 3\alpha \Delta T \]

\[ C_v = \left( \frac{d Q}{d T} \right)_v = \left( \frac{d U}{d T} \right)_v \]

\[ C_p = \left( \frac{d Q}{d T} \right)_p \]

\[ \left( \frac{d Q}{dt} \right) = -\kappa A \frac{d T}{dx} \]

\[ Q = mL \]

\[ |S| = \sigma \varepsilon T^4 \]

\[ dU = dQ - dW \]

\[ dW = PdV \]

\[ \bar{f}(s) = \frac{\int ds \ f(s) \ N(s)}{\int ds \ N(s)} \]

\[ N = \int ds \ N(s) \]

\[ \bar{e} \approx \frac{1}{2} k_B T \]

\[ C_p = C_v + Nk_B \]

\[ \gamma = \frac{C_p}{C_v} \]

\[ T(^°F) = (\frac{9}{5})T(°C) + 32°F \]

\[ S_B - S_A = \int_{A}^{B} \frac{d Q}{T} \]

\[ dU = TdS - PdV \]

\[ N(v) = 4\pi N \left( \frac{m}{2\pi k_B T} \right)^{3/2} \nu^2 e^{\frac{mv^2}{2k_B T}} \]

\[ \eta = \frac{|W_{\text{TOTAL}}|}{|Q_{\text{IN}}|} = 1 - \frac{|Q_{\text{OUT}}|}{|Q_{\text{IN}}|} \]
Electromagnetic Waves.

<table>
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<th>$f$</th>
<th>$\lambda$</th>
<th>Physical Manifestation</th>
</tr>
</thead>
<tbody>
<tr>
<td>60 Hz</td>
<td>5000 km</td>
<td>Earth - Power Grid</td>
</tr>
<tr>
<td>$10^4$ Hz (MHz)</td>
<td>300 m</td>
<td>Radio Waves, AM</td>
</tr>
<tr>
<td>$10^5$ Hz</td>
<td>3 m</td>
<td>Radio Waves, FM</td>
</tr>
<tr>
<td>$10^9$ Hz (GHz)</td>
<td>0.3 m</td>
<td>Microwaves</td>
</tr>
<tr>
<td>$10^{12}$ Hz</td>
<td>300 µm</td>
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<td>$10^{15}$ Hz</td>
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<td>$10^{16}$ Hz</td>
<td>30 Å</td>
<td>Ultraviolet</td>
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</tr>
<tr>
<td>$10^{20}$ Hz</td>
<td>30 femtometers</td>
<td>Gamma Rays</td>
</tr>
</tbody>
</table>

Nuclei
Fig. 7.9  The index of refraction (top) and absorption coefficient (bottom) for liquid water as a function of linear frequency. Also shown as abscissas are an energy scale (arrows) and a wavelength scale (vertical lines). The visible region of the frequency spectrum is indicated by the vertical dashed lines. The absorption coefficient for sea water is indicated by the dashed diagonal line at the left. Note that the scales are logarithmic in both directions.
Electromagnetic Waves, Oscillatory Solutions

Let \( \mathbf{E} = \hat{\mathbf{e}} E_0 e^{i(k \cdot \mathbf{r} - \omega t)} \)

where \( \hat{\mathbf{e}} \) unit vector that indicates the direction of electric field

\[
\mathbf{r} = k_x \hat{x} + k_y \hat{y} + k_z \hat{z}
\]

\[
\mathbf{k} \cdot \mathbf{r} = k_x x + k_y y + k_z z.
\]

Does this satisfy the wave equation? \( \frac{\partial^2 \mathbf{E}}{\partial t^2} = c^2 \nabla^2 \mathbf{E} \quad c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \)

\[
\frac{\partial^2 \mathbf{E}}{\partial t^2} = \hat{\mathbf{e}} E_0 e^{i(k \cdot \mathbf{r} - \omega t)} (-i\omega)^2 = -\omega^2 \mathbf{E}
\]

\[
\nabla^2 \mathbf{E} = \hat{\mathbf{e}} E_0 e^{i(k \cdot \mathbf{r} - \omega t)} \left[ -k_x^2 - k_y^2 - k_z^2 \right] = -k^2 \mathbf{E}
\]

and \( \mathbf{E} \) will be a solution if \( \omega^2 = c^2 k^2 \) (or \( \omega = ck \)).

Since there are no charges or currents, \( \nabla \cdot \mathbf{E} = 0 \) and so

\[
\frac{\partial}{\partial x} E_x + \frac{\partial}{\partial y} E_y + \frac{\partial}{\partial z} E_z = 0
\]

\[
E_0 e^{i(k \cdot \mathbf{r} - \omega t)} \left[ \epsilon_x k_x + \epsilon_y k_y + \epsilon_z k_z \right] = 0
\]

\( \hat{\mathbf{e}} \cdot \mathbf{k} = 0 \) \( \implies \hat{\mathbf{e}} \) must be perpendicular to \( \mathbf{k} \). (Not all directions of \( \hat{\mathbf{e}} \) and \( \mathbf{k} \) are possible).

Now the corresponding magnetic field can be calculated from

\[
\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} =
\]

\[
= -i \left[ \left( \frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} \right) \hat{x} + \left( \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} \right) \hat{y} + \left( \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} \right) \hat{z} \right]
\]

\[
E_0 \hat{\mathbf{e}} e^{i(k \cdot \mathbf{r} - \omega t)}
\]
\[
\frac{dB}{dt} = -i \left( k \times \hat{e} \right) E_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)} \quad \text{integrating with respect to } t
\]

\[
B = \frac{1}{\omega} \left( k \times \hat{e} \right) E_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)} \quad \text{but } \mathbf{k} = k \hat{k} = \omega \hat{k}
\]

\[
B = \left( \frac{E_0}{c} \right) \left( k \times \hat{e} \right) e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}
\]

\[
B \text{ is always perpendicular to } E \text{ and has a magnitude that is the magnitude of } E \text{ divided by } c! \quad B = \frac{E}{c}
\]

Now what about the Poynting vector \( S = \frac{1}{\mu_0} (E \times B) \) using the real parts.

\[
E = E_0 \hat{e} \cos(k \cdot \mathbf{r} - \omega t) \quad B = \frac{E_0}{c} \left( k \times \hat{e} \right) \cos(k \cdot \mathbf{r} - \omega t)
\]

\[
S = \frac{1}{\mu_0} (E \times B) = \frac{1}{\mu_0} \frac{E_0^2}{c} \left[ \hat{e} \times \left( k \times \hat{e} \right) \right] \cos^2(k \cdot \mathbf{r} - \omega t)
\]

\[
= \frac{E_0^2}{\mu_0 c} \left[ \left( \hat{e} \cdot \hat{e} \right) \hat{k} - (\hat{e} \cdot \hat{k}) \hat{e} \right] \cos^2(k \cdot \mathbf{r} - \omega t)
\]

But since \( \hat{e} \cdot \hat{k} = 0 \)

\[
S = \left[ \frac{E_0}{\mu_0 c} \cos^2(k \cdot \mathbf{r} - \omega t) \right] \hat{k}
\]

\( \Delta \) The power flows in the direction of \( \hat{k} \)!
In general one can decompose $\mathbf{E}$ into two components which are both perpendicular to $\mathbf{k}$.

(Two polarizations.)

\[ S = \frac{E_0^2}{\mu_0} e^{-\cos^2 (k \cdot r - \omega t)}. \]