Homework # 4  Physics 412 - Spring 2015  Due Friday, February 20, 2015

Read:  B&J  Chapters 3, 5

20. For the potential \( V[x] = \begin{cases} 
V_1 & \text{for } x < 0 \\
0 & \text{for } 0 < x < a \\
V_0 & \text{for } x > a 
\end{cases} \)

assume that \( E > V_1 > V_0 > 0 \) and the particles are incident from the right with flux \( J_0 \).

(a) Determine the transmission coefficient
(b) Is the transmission coefficient the product of the transmission coefficients for the two independent potential steps? If not, discuss why not.
(c) For what values of \( a \) is the transmission coefficient equal to one?

21. For the same potential, \( V[x] = \begin{cases} 
0 & \text{for } x < 0 \\
V_0 & \text{for } 0 < x < a \\
V_1 & \text{for } x > a 
\end{cases} \)

assume that \( V_1 > E > V_0 > 0 \) and particles are incident from the right with flux \( J_0 \).

By using the solution found in Prob. 20 and changing the appropriate constants, determine the reflection coefficient. Is the value physically reasonable.

22. Consider a particle of mass \( m \) moving in a potential \( V[x] = \begin{cases} 
0 & \text{for } x \leq 0 \\
V_0 > 0 & \text{for } 0 < x < a \\
\infty & \text{for } x \geq a 
\end{cases} \)

(a) Assume particles are incident from the left with energy \( E < V_0 \) and incident flux \( J_0 \). Determine the wavefunction in the three regions \( x \leq 0 \), \( 0 < x < a \) and \( x \geq a \).
(b) Determine the reflection coefficient. Is the value physically reasonable.
22. Consider a potential consisting of a delta function at the origin \( V[x] = \alpha \, V_0 \, \delta[x] \).
(a) What conditions must the wavefunction satisfy at \( x = 0 \).
(b) Determine the transmission and reflection coefficients for particles incident from the left with energy \( E > 0 \).
(c) Does the answer in (b) depend on the sign of \( \alpha \)?
(d) Plot the dependence of the transmission coefficient as a function of the energy relative to the potential energy scale, \( \varepsilon = E / V_0 \), for different values of the scaled parameter, \( v = V_0 \sqrt{\frac{\hbar^2}{2 \, m \, a^2}} \).

23. (This problem is best done in *Mathematica* or equivalent program). A particle moves in the potential \( V[x] = \frac{\hbar^2}{2 \, m} \left( -\frac{4}{225} \, \text{Sinh}^2[x] \cdot \frac{2}{5} \, \text{Cosh}[x] \right) \).
(a) Sketch \( V[x] \) and locate the position of the two minima.
(b) Show that \( \psi[x] = (1 + \text{Cosh}[x]) \, e^{\left( \frac{2}{15} \, \text{Cosh}[x] \right)} \) is a solution of the time-independent Schrödinger equation for the particle (operate the Hamiltonian on \( \psi[x] \) and show that it gives an energy times \( \psi[x] \).
(c) Find the corresponding energy level and indicate its position on the sketch of \( V[x] \)
(d) Sketch the wavefunction and show that is has the proper behavior at the classical turning points and in the classically forbidden regions.