QUANTUM NUMBERS

We have assumed circular orbits

Then for hydrogen

\[ E_n = -\frac{1}{n^2}13.6eV \quad \text{and} \quad L = n\frac{h}{2\pi} \]

If you know n you know both energy and angular momentum

Only one quantum number.
Sommerfeld in 1915 considered elliptical orbits for hydrogen atoms with \( n \) all same energy.

But angular momentum will be different –
- maximum for circular orbit
- minimum for straight line orbit (zero)

Need another quantum number

Have **principal quantum number**

\[ n = 1, 2, 3, 4, 5, \ldots \]
Now we introduce

**angular momentum quantum number**

\[ l = 0, 1, 2, 3, \ldots \ldots (n - 1) \]

The third quantum number is needed because the electron in a closed orbit is a current loop.

A current loop gives a magnetic moment.

Stern-Gerlach Experiment showed magnetic moments only in specific directions.

Introduce the third quantum number

**magnetic quantum number**

\[ m_l = -l, \ldots \ldots -1, 0, 1, \ldots \ldots +l \]
The electron in orbit has a magnetic moment.

For this another quantum number is needed.

**spin quantum number**

\[ m_s = -\frac{1}{2}, +\frac{1}{2} \]

With the four quantum numbers you can describe the complete state of the atom.
A system is used to identify the state of the atom.

For an electron in an atom with $l=0$

is said to be in an $s$ state.

For an electron in an atom with $l=1$

is said to be in an $p$ state.

For an electron in an atom with $l=2$

is said to be in an $d$ state.

For an electron in an atom with $l=3$

is said to be in an $e$ state.

Thus for

\[
\begin{align*}
l &= 0 \quad & 1 \quad & 2 \quad & 3 \quad & 4 \quad & 5 \\
state &= s \quad p \quad d \quad e \quad f \quad g
\end{align*}
\]
PAULI EXCLUSION PRINCIPLE

For Hydrogen – only one electron.

Other elements have more electrons

PAULI proposed rule that explains chemical behavior of elements.

IN ANY ATOM NO TWO ELECTRONS CAN HAVE THE SAME SET OF QUANTUM NUMBERS.
The quantum numbers and their relationship needed to explain the experiments are:

\[ n = 1, 2, 3, 4, 5, \ldots \]

\[ l = 0, 1, 2, 3, \ldots (n - 1) \]

\[ m_l = -l, \ldots -1, 0, 1, \ldots +l \]

\[ m_s = -\frac{1}{2}, +\frac{1}{2} \]

These relationships and the Pauli Principle will give:

Hydrogen (1 electron)

\begin{align*}
\text{electron state} & \quad n=1, \ l=0, \ m_l=0, \ m_s = -\frac{1}{2} \\
\end{align*}

Helium (2 electrons)

\begin{align*}
\text{1\textsuperscript{st} electron} & \quad n=1, \ l=0, \ m_l=0, \ m_s = -\frac{1}{2} \\
\text{2\textsuperscript{nd} electron} & \quad n=1, \ l=0, \ m_l=0, \ m_s = +\frac{1}{2} \\
\end{align*}
Lithium (3 electrons)

1st electron \( n=1, l=0, m_l=0, m_s= -\frac{1}{2} \)
2nd electron \( n=1, l=0, m_l=0, m_s= +\frac{1}{2} \)
3rd electron \( n=2, l=0, m_l=0, m_s= -\frac{1}{2} \)

Beryllium (4 electrons)

1st electron \( n=1, l=0, m_l=0, m_s= -\frac{1}{2} \)
2nd electron \( n=1, l=0, m_l=0, m_s= +\frac{1}{2} \)
3rd electron \( n=2, l=0, m_l=0, m_s= -\frac{1}{2} \)
4th electron \( n=2, l=0, m_l=0, m_s= +\frac{1}{2} \)

etc.

Review Table 8.1 in Thornton and Rex