ATOMIC MODELS

Models are formulated to fit the available data.

1900

Atom was known to have certain size.

Atom was known to be neutral.

Atom was known to give off electrons.

THOMPSON MODEL

To satisfy the above conditions Thompson proposed

Atom made of positive material with enough electrons embedded to make it neutral.
RUTHERFORD

Made measurements of alpha particles scattered from gold:

Found

1. Most alpha particles passed through gold without being scattered.

2. Some alpha particles were scattered through large scattering angles.

Data could not be explained by Thompson Model of Atom.

1. How could alpha particles pass through positive mass?

2. The force between charges was known to be

\[ F = \frac{1}{4\pi\varepsilon_0} \frac{q_1q_2}{r^2} \]
With the known charges and this force how could alpha particles be scattered through large angles?

RUTHERFORD MODEL

Small nucleus – all positive mass contained in small center part.

Electrons in space surrounding nucleus.

SCALE MODEL

Nucleus → .
Edge of Atom →
Radius = 1 mm
10 m
LINE SPECTRA

FIRST CONSIDER WHITE LIGHT THROUGH DIFFRACTION GRATING

WHITE LIGHT $\Rightarrow$ GRATING $\Rightarrow$ RAINBOW COLORS
BUT WITH HYDROGEN GAS AS SHOWN:

FIG 3.6  PAGE 93, T & R
THE ONLY WAVELENGTHS PRESENT IN VISIBLE ARE

657 nm (red)

486 nm (blue-green)

434 nm (violet)

410 nm (violet)

397 nm

Closely spaced lines out to

365 nm (series limit)
Balmer found that he could predict the wavelength of these lines with one equation:

\[
\frac{1}{\lambda} = R \left[ \frac{1}{2^2} - \frac{1}{n^2} \right]
\]

where \( R = 1.096776 \times 10^{-7} \, m^{-1} \)

and \( n = 3, 4, 5, \ldots \ldots \text{etc.} \)
The Balmer lines are in the visible

There are other lines:

One set in the ultraviolet

Three sets in the infrared

All wavelengths can be determined by one equation:

\[
\frac{1}{\lambda} = R \left[ \frac{1}{n_1^2} - \frac{1}{n_2^2} \right]
\]

RYDBERG EQUATION
WHERE

\[ n_1 = 1 \text{ and } n_2 = 2,3,4, \ldots \] gives the uv series

\[ n_1 = 2 \text{ and } n_2 = 3,4,5, \ldots \] gives the visible series

\[ n_1 = 3 \text{ and } n_2 = 4,5,6, \ldots \] gives the 1st IR series

\[ n_1 = 4 \text{ and } n_2 = 5,6,7, \ldots \] gives the 2nd IR series

\[ n_1 = 5 \text{ and } n_2 = 6,7,8, \ldots \] gives the 3rd IR series
The series have names for the physicists who did the early work in that part of the spectrum

For $n_1 = 1$ the series is named for Lyman

For $n_1 = 2$ the series is named for Balmer

For $n_1 = 3$ the series is named for Paschen

For $n_1 = 4$ the series is named for Brackett

For $n_1 = 5$ the series is named for Pfund
THE BOHR MODEL OF THE ATOM

Bohr proposed a modification of the Rutherford model that fit the data – all data before and including the Rutherford scattering data.

The Bohr Atom

Postulates:

1. Only orbits allowed are where angular momentum is integral multiple of

\[
\frac{h}{2\pi}
\]

2. No atom radiates energy as long as electron is in one of the orbital states.
DERIVATION OF RYDBERG EQUATION

(thus validation of Bohr Model)

Angular momentum = \( mvr \)

\( m = \) mass of electron

\( v = \) velocity of electron in orbit

\( r = \) radius of orbit

From 1\(^{\text{st}}\) postulate

\[
mv r = n \frac{h}{2\pi} \quad n = 1, 2, 3, \ldots
\]

Solving for \( r \)

\[
r = n \frac{h}{2\pi mv}
\] (1)
Electron in orbit accelerates toward center

\[ a = -\frac{v^2}{r} \]

The force causing the acceleration is

\[ F = \frac{-1}{4\pi\varepsilon_0} \frac{e^2}{r^2} \]

and
\[ F = ma \]

So

\[ \frac{-1}{4\pi\epsilon_0} \frac{e^2}{r^2} = m \left( -\frac{v^2}{r} \right) \]

Solving for \( v \)
\[ v = \frac{e}{\sqrt{4\pi \varepsilon_0 m r}} \] (2)

Put this in Eq. 1 above

Solve for \( r \)

\[ r_n = n^2 \frac{\hbar^2 \varepsilon_0}{\pi me^2} \]

Subscript \( n \) to remind one \( r \) for each \( n \)!
For \( n = 1 \)

\[
r_1 = \frac{\hbar^2 \varepsilon_0}{\pi \alpha e^2} = 5.0 \times 10^{-11} m \equiv a_0
\]

\( a_0 \) is the Bohr orbit for the hydrogen atom in the lowest energy state.

Then the other orbits

\[
r_n = n^2 a_0
\]
Bohr model is nucleus with electrons in only specific allowed orbits!
ENERGY OF THE ATOM

Total energy is

\[ KE + PE \]

\[
KE = \frac{1}{2}mv^2
\]

\[
PE = \frac{-e^2}{4\pi\varepsilon_0 r}
\]

and we found \( v \)
\[ v = \frac{e}{\sqrt{4 \pi \epsilon_0 mr}} \]

So

\[ \text{TotalEnergy} = E = \frac{1}{2} m \left[ \frac{e}{\sqrt{4\pi\epsilon_0 mr}} \right]^2 + \frac{-e^2}{4\pi\epsilon_0 r} \]

\[ = -\frac{e^2}{8\pi\epsilon_0 r} \]

Or

\[ E_n = -\frac{e^2}{8\pi\epsilon_0 r_n} \]
and
\[ r_n = n^2 \frac{h^2 \epsilon_0}{\pi m e^2} \]

Putting this in Total Energy equation

\[ E_n = -\frac{m e^4}{8 \epsilon_0 h^2} \frac{1}{n^2} \]

where \( n = 1, 2, 3, \ldots \)

For:

\( n=1 \) \( E_1 = -21.76 \times 10^{-19} \text{ J} = -13.6 \text{ eV} \)
\( n=2 \) \( E_2 = -5.43 \times 10^{-19} \text{ J} = -3.39 \text{ eV} \)
\( n=3 \) \( E_3 = -2.41 \times 10^{-19} \text{ J} = -1.51 \text{ eV} \)
\( n=4 \) \( E_4 = -1.36 \times 10^{-19} \text{ J} = -0.85 \text{ eV} \)
Plot these values on a vertical scale to get Figure 4.16 on page 145 of *Thornton and Rex*
DERIVATION OF RYDBERG EQUATION

Consider any two energy levels

\[ E_i \text{ with } n_i \]

and

\[ E_j \text{ with } n_j \]

If the atom is in \( n_i \) state with total energy \( E_i \) and goes to the \( n_j \) state with total energy \( E_j \), the atom will have less total energy.

The difference in energy will be

\[ \Delta E = E_i - E_j \]

The energy is carried off by a photon with energy
\[ hf = E_i - E_j \]

Or since

\[ f = \frac{c}{\lambda} \]

\[ \frac{1}{\lambda} = \frac{f}{c}(E_i - E_j) \]
and

\[ E_i = -\frac{me^4}{8\varepsilon_0 h^3} \frac{1}{n_i^2} \]

\[ E_j = -\frac{me^4}{8\varepsilon_0 h^3} \frac{1}{n_j^2} \]

So

\[ \frac{1}{\lambda} = \frac{me^4}{8\varepsilon_0^2 ch^3} \left( \frac{1}{n_j^2} - \frac{1}{n_i^2} \right) \]

If
\[
\frac{m e^4}{8 \varepsilon_0^2 c h^3} = R = 1.097 \times 10^7 \text{ m}^{-1}
\]
(and it does)

Then for

\[n_j = 1\] We have the Lyman Equation

\[n_j = 2\] We have the Balmer Equation

\[n_j = 3\] We have the Paschen Equation

\[n_j = 4\] We have the Brackett Equation

\[n_j = 5\] We have the Pfund Equation

STUDY FIGURE 4.16 ON PAGE 145 IN

THORNTON AND REX!!