THE COMPTON EFFECT

Planck:
Radiation must be emitted in bundles of energy

Einstein:
Once radiated remains bundle of energy (photon) until absorbed

Idea that photons are “particles” is shown conclusively in Compton Experiment

Consider an electron at rest – it has no momentum and its energy, $E_1$, is only its rest energy $mc^2$.

The electron is hit by a photon with wavelength $\lambda_0$ and energy $hf_0$. 
(b) Quantum model

$f_0, \lambda_0$

$p_e$

Recoiling particle

$\phi$

$\theta$

Scattered particle

$f', \lambda'$

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The electron is scattered above the axis at an angle $\varphi$ and the photon is scattered below the axis by angle $\theta$.

Before the interaction:

The photon has momentum in the $x$ direction

\[
\frac{h}{\lambda}
\]

and energy

\[
\frac{hc}{\lambda}
\]

The electron has zero momentum and energy

\[mc^2\]
After the interaction:

The electron has energy

\[ E = \left( p_e^2 c^2 + m^2 c^4 \right)^{\frac{1}{2}} \]

The electron has momentum directed at an angle \( \varphi \) above the axis equal to \( p_e \).

The new photon has energy

\[ \frac{hc}{\lambda'} \]

and momentum directed at an angle \( \theta \) below the axis

\[ \frac{h}{\lambda'} \]
Conservation of energy gives

\[
\frac{hc}{\lambda} + m_e c^2 = \frac{hc}{\lambda'} + \left( p_e^2 c^2 + m_e^2 c^4 \right)^{\frac{1}{2}}
\]

or

\[
\frac{hc}{\lambda} - \frac{hc}{\lambda'} + m_e c^2 = \left( p^2 c^2 + m_e^2 c^4 \right)^{\frac{1}{2}}
\]  \hspace{1cm} (A)

For conservation of momentum in the x direction

\[
\frac{h}{\lambda} = \frac{h}{\lambda'} \cos \theta + p_e \cos \phi
\]  \hspace{1cm} (B)
For conservation of momentum in the y direction

\[ 0 = \frac{h}{\boldsymbol{\lambda}} \sin \theta - p_e \sin \phi \]  
\[ (C) \]

Use equations (B) and (C) to eliminate \( \phi \)

Solve for \( p \cos \phi \) in (B) and \( p \sin \phi \) in (C) and square

\[ p_e^2 \cos^2 \phi = \frac{h^2}{\boldsymbol{\lambda}^2} - \frac{2h^2 \cos \theta}{\boldsymbol{\lambda} \hat{\lambda}} + \frac{h^2 \cos^2 \theta}{\boldsymbol{\lambda}^2} \]

\[ p_e^2 \sin^2 \phi = \frac{h^2 \sin^2 \theta}{\hat{\lambda}^2} \]
Add to get

\[ p_e^2 = h^2 \left( \frac{1}{\lambda^2} - \frac{2 \cos \theta}{\lambda \lambda'} + \frac{1}{\lambda'^2} \right) \]  \hspace{1cm} (D)

Now square equation (A)

\[ p_e^2 c^2 = \left[ \left( \frac{hc}{\lambda} \right)^2 - 2 \left( \frac{hc}{\lambda} \right) \left( \frac{hc}{\lambda'} \right) + \left( \frac{hc}{\lambda'} \right)^2 \right] + \left[ \left( \frac{2hc}{\lambda} \right) - 2 \left( \frac{hc}{\lambda'} \right) \right] mc^2 \]

or

\[ p_e^2 c^2 = \left( \frac{hc}{\lambda} - \frac{hc}{\lambda'} \right)^2 + 2hc \left( \frac{1}{\lambda} - \frac{1}{\lambda'} \right) mc^2 \]

Solve for \( p_e^2 \) and set equal to the right hand side of Equation (D)
\[
2mhc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) + h^2\left(\frac{1}{\lambda^2} - \frac{2}{\lambda \lambda'} + \frac{1}{\lambda'^2}\right) = h^2\left(\frac{1}{\lambda^2} - \frac{2 \cos \theta}{\lambda \lambda'} + \frac{1}{\lambda'^2}\right)
\]

Cancel terms and multiply by \(\frac{\lambda \lambda'}{h}\)

To get

\[
m c \left(\lambda - \lambda'\right) - h = -h \cos \theta
\]

or finally

\[
\Delta \lambda = \lambda' - \lambda = \frac{h}{m_e c} (1 - \cos \theta)
\]
Intensity

\( \theta = 0^\circ \)
Primary

\( \lambda_0 \rightarrow \lambda \)

\( \theta = 45^\circ \)

\( \lambda_0 \lambda' \rightarrow \lambda \)

\( \theta = 90^\circ \)

\( \lambda_0 \rightarrow \lambda' \rightarrow \lambda \)

\( \theta = 135^\circ \)

(b)