TIME INDEPENDENT

SCHROEDINGER EQUATION

PROBLEMS WHERE THE

POTENTIAL $V(x,t)$ IS ONLY A

FUNCTION OF $x$, ie. $V(x)$

THEN SCHROEDINGER EQUATION

\[
-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x,t)}{\partial x^2} + V(x)\Psi(x,t) = i\hbar \frac{\partial \Psi(x,t)}{\partial t}
\]

SOLUTION IS

$\Psi(x,t)$ WAVE FUNCTION
SOLVE BY SEPARATION OF VARIABLES

ASSUME SOLUTION

\[ \Psi(x,t) = \psi(x)\Phi(t) \]

\( \Psi(x,t) \) CALLED WAVE FUNCTION

\( \psi(x) \) CALLED EIGEN FUNCTION

\( \Phi(t) \) CALLED TIME DEPENDENCE OF WAVE FUNCTION
PUT INTO WAVE EQUATION

\[-\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x) \Phi(t)}{\partial x^2} + V(x)\psi(x)\Phi(t) = i\hbar \frac{\partial \psi(x)\Phi(t)}{\partial t}\]

OR

\[-\frac{\hbar^2}{2m} \Phi(t) \frac{\partial^2 \psi(x)}{\partial x^2} + V(x)\psi(x)\Phi(t) = i\hbar \psi(x) \frac{\partial \Phi(t)}{\partial t}\]

NOW TOTAL DIFFERENTIALS
CHANGE

PARCIAL $\partial$

TO

TOTAL $d$

$$- \frac{\hbar^2}{2m} \Phi(t) \frac{d^2\psi(x)}{dx^2} + V(x)\psi(x)\Phi(t) = i\hbar \psi(x) \frac{d\Phi(t)}{dt}$$

REARRANGE – ALL $x$ TERMS ON LEFT

ALL $t$ TERMS ON RIGHT
DIVIDE BY

\[ \psi(x)\Phi(t) \]

\[- \frac{\hbar^2}{2m} \frac{\Phi(t)}{\psi(x)\Phi(t)} \frac{d^2\psi(x)}{dx^2} + \frac{V(x)\psi(x)\Phi(t)}{\psi(x)\Phi(t)} = i\hbar \frac{\psi(x)}{\psi(x)\Phi(t)} \frac{d\Phi(t)}{dt} \]

AND

\[- \frac{\hbar^2}{2m} \frac{1}{\psi(x)} \frac{d^2\psi(x)}{dx^2} + V(x) = i\hbar \frac{1}{\Phi(t)} \frac{d\Phi(t)}{dt} \]
LEFT IS FUNCTION OF x **ONLY**

RIGHT IS FUNCTION OF t **ONLY**

SO EACH SIDE IS EQUAL TO CONSTANT

CALL IT C
\[-\frac{\hbar^2}{2m} \frac{1}{\psi(x)} \frac{d^2 \psi(x)}{dx^2} + V(x) = C\]  \hspace{1cm} 1.

AND

\[i\hbar \frac{1}{\Phi(t)} \frac{d\Phi(t)}{dt} = C\]  \hspace{1cm} 2.

WE WILL SOLVE EQUATION No. 2

REARRANGE
\[
\frac{d\Phi(t)}{dt} - \frac{C}{i\hbar} \Phi(t) = 0
\]

2\textsuperscript{nd} TERM MULT TOP AND BOTTOM

BY \( i \)

\[
\frac{d\Phi(t)}{dt} + \frac{iC}{\hbar} \Phi(t) = 0
\]

C IS JUST A CONSTANT

LET IT EQUAL ANOTHER CONSTANT
\[ C = \hbar \omega \]

\[ \frac{d\Phi(t)}{dt} + \frac{i\hbar \omega}{\hbar} \Phi(t) = 0 \]

\[ \frac{d\Phi(t)}{dt} + i\omega \Phi(t) = 0 \]

**STANDARD DIFFERENTIAL EQUATION WITH SOLUTION**

\[ \Phi(t) = e^{-i\omega t} \]
REMEMBER

\[ E = \hbar \omega \]

OR

\[ \omega = \frac{E}{\hbar} \]

SO

\[ \Phi(t) = e^{-\frac{iEt}{\hbar}} \]
ALSO

\[ \hbar = \frac{h}{2\pi} \quad \text{and} \quad \omega = 2\pi f \]

SO

\[ \hbar \omega = \frac{h}{2\pi} 2\pi f = hf = ENERGY \]

USE

\[ C=E \quad \text{IN EQUATION No. 1} \]

\[ -\frac{\hbar^2}{2m} \frac{1}{\psi(x)} \frac{d^2\psi(x)}{dx^2} + V(x) = E \]
REARRANGE

TO GET

\[- \frac{\hbar^2}{2m} \frac{d^2 \psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)\]

THIS IS TIME INDEPENDENT

SCHROEDINGER WAVE EQUATION

WITH SOLUTION
\( \psi(x) \)

THE EIGENFUNCTION
NOW THE WAVE FUNCTION IS

$$\Psi(x, t) = \psi(x) \Phi(t)$$

WHERE

$$\psi(x)$$ IS SOLUTION OF TIME
INDEPENDENT WAVE EQUATION

AND

$$\Phi(t) = e^{-i \frac{E}{\hbar} t}$$
OR FINALLY

\[ \Psi(x, t) = \psi(x) e^{-i \frac{E}{\hbar} t} \]
THEREFORE TO WORK ANY PROBLEM WHERE THE POTENTIAL IS INDEPENDENT OF TIME

1. PUT POTENTIAL IN TISE

2. SOLVE FOR EIGENFUNCTION

3. MULTIPLY EIGENFUNCTION BY TIME DEPENDENCE

\[
\Phi(t) = e^{-i\frac{E}{\hbar}t}
\]

TO GET WAVE FUNCTION

\[
\Psi(x, t)
\]
FROM WAVE FUNCTION CAN FIND ALL THERE IS TO MEASURE ABOUT PARTICLE BY

1. FORCING WAVE FUNCTION TO BE WELL BEHAVED (SINGLE VALUED, FINITE, CONTINUOUS, etc.)

2. NORMALIZED (IF NECESSARY)

THEN USING IT IN THE PROCEDURES