27.1. **IDENTIFY** and **SET UP:** Apply Eq.(27.2) to calculate \( \mathbf{F} \). Use the cross products of unit vectors from Section 1.10.

**EXECUTE:**

(a) \( \mathbf{\bar{B}} = (1.40 \text{ T})\hat{i} \)

\[
\mathbf{F} = q\mathbf{v} \times \mathbf{\bar{B}} = (-1.24 \times 10^{-8} \text{ C})(1.40 \text{ T})\left[ (4.19 \times 10^4 \text{ m/s})\hat{i} \times \hat{i} - (3.85 \times 10^4 \text{ m/s})\hat{j} \times \hat{i} \right]
\]

\[
\hat{i} \times \hat{i} = 0, \hat{j} \times \hat{i} = -\hat{k}
\]

\[
\mathbf{F} = (-1.24 \times 10^{-8} \text{ C})(1.40 \text{ T})(-3.85 \times 10^4 \text{ m/s})\hat{k} = (-6.68 \times 10^{-4} \text{ N})\hat{k}
\]

**EVALUATE:** The directions of \( \mathbf{v} \) and \( \mathbf{\bar{B}} \) are shown in Figure 27.1a.

The right-hand rule gives that \( \mathbf{v} \times \mathbf{\bar{B}} \) is directed out of the paper (+z-direction). The charge is negative so \( \mathbf{F} \) is opposite to \( \mathbf{v} \times \mathbf{\bar{B}} \);

![Figure 27.1a](image)

\( \mathbf{F} \) is in the \(-z\)-direction. This agrees with the direction calculated with unit vectors.

(b) **EXECUTE:** \( \mathbf{\bar{B}} = (1.40 \text{ T})\hat{k} \)

\[
\mathbf{F} = q\mathbf{v} \times \mathbf{\bar{B}} = (-1.24 \times 10^{-8} \text{ C})(1.40 \text{ T})\left[ (4.19 \times 10^4 \text{ m/s})\hat{i} \times \hat{k} - (3.85 \times 10^4 \text{ m/s})\hat{j} \times \hat{k} \right]
\]

\[
\hat{i} \times \hat{k} = -\hat{j}, \hat{j} \times \hat{k} = \hat{i}
\]

\[
\mathbf{F} = (-7.27 \times 10^{-4} \text{ N})(-\hat{j}) + (6.68 \times 10^{-4} \text{ N})\hat{i} = \left[ (6.68 \times 10^{-4} \text{ N})\hat{i} + (7.27 \times 10^{-4} \text{ N})\hat{j} \right]
\]

**EVALUATE:** The directions of \( \mathbf{v} \) and \( \mathbf{\bar{B}} \) are shown in Figure 27.1b.

The direction of \( \mathbf{F} \) is opposite to \( \mathbf{v} \times \mathbf{\bar{B}} \) since \( q \) is negative. The direction of \( \mathbf{F} \) computed from the right-hand rule agrees qualitatively with the direction calculated with unit vectors.

![Figure 27.1b](image)
27.8. IDENTIFY and SET UP: \[ \vec{F} = q \vec{v} \times \vec{B} = q B \bigl[ v_x (\hat{i} \times \hat{k}) + v_y (\hat{j} \times \hat{k}) + v_z (\hat{k} \times \hat{k}) \bigr] = q B \bigl[ v_x (\hat{j}) + v_y (\hat{i}) \bigr]. \]

EXECUTE: (a) Set the expression for \( \vec{F} \) equal to the given value of \( \vec{F} \) to obtain:

\[ v_x = \frac{F_x}{-q B_z} = \frac{(-5.60 \times 10^{-7} \text{ C})(-1.25 \text{ T})}{-5.60 \times 10^{-7} \text{ N}} = -106 \text{ m/s} \]

\[ v_y = \frac{F_y}{q B_z} = \frac{-(3.40 \times 10^{-7} \text{ N})}{-5.60 \times 10^{-7} \text{ N}} = -48.6 \text{ m/s}. \]

(b) \( v_z \) does not contribute to the force, so it is not determined by a measurement of \( \vec{F} \).

(c) \( \hat{v} \cdot \vec{F} = v_x F_x + v_y F_y + v_z F_z = \frac{F_x}{-q B_z} F_x + \frac{F_y}{q B_z} F_y = 0; \ \theta = 90^\circ. \]

EVALUATE: The force is perpendicular to both \( \hat{v} \) and \( \vec{B} \), so \( \vec{B} \cdot \vec{F} \) is also zero.

27.12. IDENTIFY: When \( \vec{B} \) is uniform across the surface, \( \Phi_B = \vec{B} \cdot A = B A \cos \phi \).

SET UP: \( \vec{A} \) is normal to the surface and is directed outward from the enclosed volume. For surface \( abcd \), \( \vec{A} = -\vec{A} \). For surface \( befc \), \( \vec{A} = -\vec{A} \). For surface \( aefd \), \( \cos \phi = 3/5 \) and the flux is positive.

EXECUTE: (a) \( \Phi_B (abcd) = \vec{B} \cdot \vec{A} = 0. \)

(b) \( \Phi_B (befc) = \vec{B} \cdot \vec{A} = -(0.128 \text{ T})(0.300 \text{ m})(0.300 \text{ m}) = -0.0115 \text{ Wb}. \)

(c) \( \Phi_B (aefd) = \vec{B} \cdot \vec{A} = BA \cos \phi = \frac{1}{3} (0.128 \text{ T})(0.500 \text{ m})(0.300 \text{ m}) = +0.0115 \text{ Wb}. \)

(d) The net flux through the rest of the surfaces is zero since they are parallel to the \( x \)-axis. The total flux is the sum of all parts above, which is zero.

EVALUATE: The total flux through any closed surface, that encloses a volume, is zero.

27.22. IDENTIFY: For motion in an arc of a circle, \( a = \frac{v^2}{R} \) and the net force is radially inward, toward the center of the circle.

SET UP: The direction of the force is shown in Figure 27.22. The mass of a proton is \( 1.67 \times 10^{-27} \text{ kg} \).

EXECUTE: (a) \( \vec{F} \) is opposite to the right-hand rule direction, so the charge is negative. \( \vec{F} = m \hat{a} \) gives

\[ qB \sin \phi = m \frac{v^2}{R} \Rightarrow \phi = 90^\circ \text{ and } v = \frac{qB R}{m} = \frac{3(1.60 \times 10^{-19} \text{ C})(0.250 \text{ T})(0.475 \text{ m})}{12(1.67 \times 10^{-27} \text{ kg})} = 2.84 \times 10^6 \text{ m/s}. \]

(b) \( F_y = q|B| \sin \phi = 3(1.60 \times 10^{-19} \text{ C})(2.84 \times 10^6 \text{ m/s})(0.250 \text{ T}) \sin 90^\circ = 3.41 \times 10^{-13} \text{ N} \).

\( w = mg = 12(1.67 \times 10^{-27} \text{ kg})(9.80 \text{ m/s}^2) = 1.96 \times 10^{-25} \text{ N} \). The magnetic force is much larger than the weight of the particle, so it is a very good approximation to neglect gravity.

EVALUATE: (c) The magnetic force is always perpendicular to the path and does no work. The particles move with constant speed.

\[
\begin{align*}
F & \\
& \text{Figure 27.22}
\end{align*}
\]
27.39. **IDENTIFY** and **SET UP**: The magnetic force is given by Eq.(27.19). \( F_i = mg \) when the bar is just ready to levitate. When \( I \) becomes larger, \( F_i > mg \) and \( F_i - mg \) is the net force that accelerates the bar upward. Use Newton's 2nd law to find the acceleration.

(a) **EXECUTE**: \( II \) \( B = mg \), \( I = \frac{mg}{IB} = \frac{0.750 \text{ kg}}{9.80 \text{ m/s}^2} \)
\( E = IR = (32.67 \text{ A})(25.0 \Omega) = 817 \text{ V} \)
(b) \( R = 2.0 \Omega \), \( I = \frac{E}{R} = \frac{816.7 \text{ V}}{2.0 \Omega} = 408 \text{ A} \)
\( F_i = II \) \( B = 92 \text{ N} \)
\( a = (F_i - mg)\) \( /m = 113 \text{ m/s}^2 \)
**EVALUATE**: \( I \) increases by over an order of magnitude when \( R \) changes to \( F_i \) \( >> mg \) and \( a \) is an order of magnitude larger than \( g \).

27.44. **IDENTIFY**: \( \tau = IAB\sin \phi \), where \( \phi \) is the angle between \( B \) and the normal to the loop.

**SET UP**: The coil as viewed along the axis of rotation is shown in Figure 27.44a for its original position and in Figure 27.44b after it has rotated \( 30.0^\circ \).

**EXECUTE**: (a) The forces on each side of the coil are shown in Figure 27.44a. \( \vec{F}_1 + \vec{F}_2 = 0 \) and \( \vec{F}_3 + \vec{F}_4 = 0 \). The net force on the coil is zero. \( \phi = 0^\circ \) and \( \sin \phi = 0 \), so \( \tau = 0 \). The forces on the coil produce no torque.
(b) The net force is still zero. \( \phi = 30.0^\circ \) and the net torque is \( \tau = (1)(1.40 \text{ A})(0.220 \text{ m})(0.350 \text{ m})(1.50 \text{ T})\sin 30.0^\circ = 0.0808 \text{ N} \cdot \text{m} \). The net torque is clockwise in Figure 27.44b and is directed so as to increase the angle \( \phi \).

**EVALUATE**: For any current loop in a uniform magnetic field the net force on the loop is zero. The torque on the loop depends on the orientation of the plane of the loop relative to the magnetic field direction.
27.57. (a) **IDENTIFY and SET UP:** The maximum radius of the orbit determines the maximum speed $v$ of the protons. Use Newton's 2nd law and $a = \frac{v^2}{R}$ for circular motion to relate the variables. The energy of the particle is the kinetic energy $K = \frac{1}{2}mv^2$.

**EXECUTE:** \[ \sum \vec{F} = m\vec{a} \text{ gives } |q|vB = m(\frac{v^2}{R}) \]

\[ v = \frac{|q|BR}{m} = \frac{(1.60 \times 10^{-19} \text{ C})(0.85 \text{ T})(0.40 \text{ m})}{1.67 \times 10^{-27} \text{ kg}} = 3.257 \times 10^7 \text{ m/s} \]

The kinetic energy of a proton moving with this speed is

\[ K = \frac{1}{2}mv^2 = \frac{1}{2}(1.67 \times 10^{-27} \text{ kg})(3.257 \times 10^7 \text{ m/s})^2 = 8.9 \times 10^{-13} \text{ J} = 5.6 \text{ MeV} \]

(b) The time for one revolution is the period \[ T = \frac{2\pi R}{v} = \frac{2\pi (0.40 \text{ m})}{3.257 \times 10^7 \text{ m/s}} = 7.7 \times 10^{-8} \text{ s} \]

(c) \[ K = \frac{1}{2}mv^2 = \frac{1}{2}m\left( \frac{|q|BR}{m} \right)^2 = \frac{1}{2}|q|^2 \frac{B^2R^2}{m} \]

Or, \[ B = \frac{\sqrt{2Km}}{|q|R} \]

$B$ is proportional to $\sqrt{K}$, so if $K$ is increased by a factor of 2 then $B$ must be increased by a factor of $\sqrt{2}$. \[ B = \sqrt{2}(0.85 \text{ T}) = 1.2 \text{ T} \]

(d) \[ v = \frac{|q|BR}{m} = \frac{(3.20 \times 10^{-19} \text{ C})(0.85 \text{ T})(0.40 \text{ m})}{6.65 \times 10^{-27} \text{ kg}} = 1.636 \times 10^7 \text{ m/s} \]

\[ K = \frac{1}{2}mv^2 = \frac{1}{2}(6.65 \times 10^{-27} \text{ kg})(1.636 \times 10^7 \text{ m/s})^2 = 8.9 \times 10^{-13} \text{ J} = 5.5 \text{ MeV} \]

the same as the maximum energy for protons.

**EVALUATE:** We can see that the maximum energy must be approximately the same as follows: From part (c), \[ K = \frac{1}{2}m\left( \frac{|q|BR}{m} \right)^2 \]

For alpha particles $|q|$ is larger by a factor of 2 and $m$ is larger by a factor of 4 (approximately). Thus $|q|^2/m$ is unchanged and $K$ is the same.
27.67. **IDENTIFY:** The force exerted by the magnetic field is given by Eq.(27.19). The net force on the wire must be zero.

**SET UP:** For the wire to remain at rest the force exerted on it by the magnetic field must have a component directed up the incline. To produce a force in this direction, the current in the wire must be directed from right to left in Figure 27.61 in the textbook. Or, viewing the wire from its left-hand end the directions are shown in Figure 27.67a.

![Figure 27.67a](image)

The free-body diagram for the wire is given in Figure 27.67b.

![Figure 27.67b](image)

**EXECUTE:**

\[ \sum F_y = 0 \]
\[ F_y \cos \theta - Mg \sin \theta = 0 \]
\[ F_y = ILB \sin \phi \]
\[ \phi = 90^\circ \text{ since } \vec{B} \text{ is perpendicular to the current direction.} \]

Thus \((ILB) \cos \theta - Mg \sin \theta = 0\) and \( I = \frac{Mg \tan \theta}{LB} \)

**EVALUATE:** The magnetic and gravitational forces are in perpendicular directions so their components parallel to the incline involve different trig functions. As the tilt angle \( \theta \) increases there is a larger component of \( Mg \) down the incline and the component of \( F_y \) up the incline is smaller; \( I \) must increase with \( \theta \) to compensate. As \( \theta \rightarrow 0, I \rightarrow 0 \) and as \( \theta \rightarrow 90^\circ, I \rightarrow \infty \).
IDENTIFY: For the loop to be in equilibrium the net torque on it must be zero. Use Eq.(27.26) to calculate the torque due to the magnetic field and use Eq.(10.3) for the torque due to the gravity force.

SET UP: See Figure 27.75a.

EXECUTE: See Figure 27.75b.

\[ \tau_{mg} = mg \sin \phi = mg(0.400 \text{ m})\sin 30.0^\circ \]

The torque is clockwise; \( \tau_{mg} \) is directed into the paper.

For the loop to be in equilibrium the torque due to \( \vec{B} \) must be counterclockwise (opposite to \( \tau_{mg} \)) and it must be that \( \tau_\phi = \tau_{mg} \). See Figure 27.75c.

\[ \vec{\tau}_\phi = \vec{\mu} \times \vec{B} \]

For this torque to be counterclockwise (\( \tau_\phi \) directed out of the paper), \( \vec{B} \) must be in the \(+y\)-direction.

\[ \tau_\phi = \mu B \sin \phi = IAB \sin 60.0^\circ \]

\[ \tau_\phi = \tau_{mg} \]

\[ m = (0.15 \text{ g/cm}) 2(8.00 \text{ cm} + 6.00 \text{ cm}) = 4.2 \text{ g} = 4.2 \times 10^{-3} \text{ kg} \]

\[ A = (0.800 \text{ m})(0.0600 \text{ m}) = 4.80 \times 10^{-3} \text{ m}^2 \]

\[ B = \frac{mg (0.0400 \text{ m})(\sin 30.0^\circ)}{IA \sin 60.0^\circ} \]

\[ B = \frac{(4.2 \times 10^{-3} \text{ kg})(9.80 \text{ m/s}^2)(0.0400 \text{ m})\sin 30.0^\circ}{(8.2 \text{ A})(4.80 \times 10^{-3} \text{ m}^2) \sin 60.0^\circ} = 0.024 \text{ T} \]

EVALUATE: As the loop swings up the torque due to \( \vec{B} \) decreases to zero and the torque due to \( mg \) increases from zero, so there must be an orientation of the loop where the net torque is zero.
27.89. **IDENTIFY and SET UP:** In the magnetic field, $R = \frac{mv}{qB}$. Once the particle exits the field it travels in a straight line. Throughout the motion the speed of the particle is constant.

**EXECUTE:**

(a) $R = \frac{mv}{qB} = \frac{(3.20 \times 10^{-11} \text{ kg})(1.45 \times 10^7 \text{ m/s})}{(2.15 \times 10^{-7} \text{ C})(0.420 \text{ T})} = 5.14 \text{ m}$.

(b) See Figure 27.89. The distance along the curve, $d$, is given by $d = R\theta = \frac{0.35 \text{ m}}{5.14 \text{ m}} \sin \theta = 0.0486 \text{ rad}$, so $\theta = 2.78^\circ = 0.0486 \text{ rad}$. $d = R\theta = (5.14 \text{ m})(0.0486 \text{ rad}) = 0.25 \text{ m}$. And $t = \frac{d}{v} = \frac{0.25 \text{ m}}{1.45 \times 10^5 \text{ m/s}} = 1.72 \times 10^{-6} \text{ s}$.

(c) $\Delta x_1 = d\tan(\theta/2) = (0.25 \text{ m})\tan(2.79^\circ/2) = 6.08 \times 10^{-3} \text{ m}$.

(d) $\Delta x = \Delta x_1 + \Delta x_2$, where $\Delta x_2$ is the horizontal displacement of the particle from where it exits the field region to where it hits the wall. $\Delta x_2 = (0.50 \text{ m})\tan 2.79^\circ = 0.0244 \text{ m}$. Therefore, $\Delta x = 6.08 \times 10^{-3} \text{ m} + 0.0244 \text{ m} = 0.0305 \text{ m}$.

**EVALUATE:** $d$ is much less than $R$, so the horizontal deflection of the particle is much smaller than the distance it travels in the $y$-direction.

![Figure 27.89](image-url)