DIRECT-CURRENT CIRCUITS

BASIC CONCEPTS

Resistors in circuits.

Kirchhoff’s Rules

Ammeters and Voltmeters

R-C Circuits
In Chapter 24 we found equivalent capacitance for combinations of capacitors.

(a) Two capacitors in series

**Capacitors in series:**
- The capacitors have the same charge $Q$.
- Their potential differences add:
  \[ V_{ac} + V_{cb} = V_{ab}. \]

\[
\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \text{ for series}
\]
(a) Two capacitors in parallel

**Capacitors in parallel:**
- The capacitors have the same potential \( V \).
- The charge on each capacitor depends on its capacitance: \( Q_1 = C_1 V, Q_2 = C_2 V \).

\[
V_{ab} = V \quad C_1 \quad Q_1 \quad C_2 \quad Q_2
\]

\[
C_{eq} = C_1 + C_2 \text{ for parallel}
\]
For resistors we need similar equations.

(a) $R_1$, $R_2$, and $R_3$ in series

The current $I$ through each $R$ is same.

The voltage $V_{ab} = V_{ax} + V_{xy} + V_{yb}$

And $V = IR$
Therefore

\[ IR_1 = IR_2 + IR_3 = IR_{\text{equivalent}} \]

Can replace the three resistors with one resistor with resistance \( R_{\text{equivalent}} \)

\[ R_{\text{equivalent}} = R_1 + R_2 + R_3 \text{ for series} \]
The total current $I$ is divided between $R_1, R_2, and R_3$

$$I = I_1 + I_2 + I_3$$
And

\[ I = \frac{V}{R} \]

Therefore

\[ \frac{V}{R_{\text{equivalent}}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \]

Or

\[ \frac{1}{R_{\text{equivalent}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \]

For Parallel.
Consider the following circuit.

What is the current through the 6 Ω resistor?
\[ I_{2\Omega} = \frac{V_{2\Omega}}{2\Omega} = \frac{12V}{2\Omega} = 6A \]

Therefore there are 6A through the 1Ω resistor.

\[ V_{1\Omega} = I_{1\Omega} 1\Omega = 6A 1\Omega = 6V \]

The \textit{emf} of the battery is

\[ \varepsilon = V_{1\Omega} + V_{2\Omega} = 6V + 12V = 18V \]

The current through the 6Ω resistor is

\[ I_{6\Omega} = \frac{\varepsilon}{6\Omega} = \frac{18V}{6\Omega} = 3A \]
Example:

What does the voltmeter read?
KIRCHHOFF’S RULES

Junction Rule

The sum of the currents into a junction is zero.

(a) Kirchhoff’s junction rule

\[ I_1 \rightarrow \quad \leftrightarrow \quad I_2 \]

\[ I_1 + I_2 \]
**Loop Rule**

The sum of the potential differences in a loop is zero.
Sign Convention

For resistors:

(b) Sign conventions for resistors

\[ +IR: \text{Travel opposite to current direction:} \]

\[ -IR: \text{Travel in current direction:} \]

For batteries:

(a) Sign conventions for emfs

\[ +\mathcal{E}: \text{Travel direction from \(-\) to \(+\):} \]

\[ -\mathcal{E}: \text{Travel direction from \(+\) to \(-\):} \]
Example:

(a)

Loop 1 (start at $a$)

$$-I_R R + \varepsilon_1 - I_1 r_1 = 0$$

Loop 2 (start at $c$)

$$+\varepsilon_1 - I_1 r_1 - I_2 r_2 - \varepsilon_2 = 0$$
Loop 3 (start at a)

\[-I_R R + \varepsilon_2 + I_2 r_2 = 0\]

Junction a

\[I_1 = I_2 + I_R\]

Junction b

\[I_2 + I_R = I_1\]
R-C CIRCUITS

(a) Capacitor initially uncharged

No current.
Then close switch.

(b) Charging the capacitor

When the switch is closed, the charge on the capacitor increases over time while the current decreases.

Apply Kirchhoff’s Loop Rule.

Across resistor potential change $IR$

Across battery potential change $\varepsilon$

Across capacitor potential change $\frac{Q}{C}$
Start at $a$.

$$-\varepsilon + \frac{Q}{C} + iR = 0$$

The current is $i = \frac{dQ}{dt}$

Put in equation.

$$-\varepsilon + \frac{Q}{C} + \frac{dQ}{dt}R = 0$$

$$\varepsilon = \frac{Q}{C} + R \frac{dQ}{dt}$$
\[
\frac{dQ}{dt} = \frac{\varepsilon - \frac{Q}{C}}{R} = \frac{C\varepsilon - Q}{RC}
\]

\[
\frac{dQ}{C\varepsilon - Q} = \frac{dt}{RC}
\]

The charge on the capacitor goes from 0 to \(Q\). Start the stopwatch when the switch is closed and time goes from 0 to \(t\).

Integrate

\[
\int_{0}^{Q} \frac{dQ}{C\varepsilon - Q} = \frac{1}{RC} \int_{0}^{t} dt
\]
\(-\ln(C\varepsilon - Q)^0_0 = \frac{t}{RC}\)

\[\ln \frac{C\varepsilon - Q}{C\varepsilon} = -\frac{t}{RC}\]

\[1 - \frac{Q}{C\varepsilon} = e^{-\frac{t}{RC}}\]

\[\frac{Q}{C\varepsilon} = 1 - e^{-\frac{t}{RC}}\]

\[Q = C\varepsilon \left(1 - e^{-\frac{t}{RC}}\right)\]
The charge on the capacitor increases exponentially with time.

\[ Q_f = C \varepsilon \]
Charge on capacitor:

\[ Q = Q_f \left(1 - e^{-\frac{t}{RC}}\right) \]

Current in circuit:

\[ i = \frac{dQ}{dt} = \frac{d}{dt} \left[ Q_f \left(1 - e^{-\frac{t}{RC}}\right)\right] \]

\[ i = Q_f \left[0 - \left(-\frac{1}{RC}\right) e^{-\frac{t}{RC}}\right] \]

\[ i = \frac{Q_f}{RC} e^{-\frac{t}{RC}} \]

\[ Q_f = C\varepsilon \]
\[ i = \frac{\varepsilon}{R} e^{-\frac{t}{RC}} \]

And \( \frac{\varepsilon}{R} = i_0 \)

\[ i = i_0 e^{-\frac{t}{RC}} \]

Define \( \tau = RC = \text{Circuit Time Constant} \)

\[ i = i_0 e^{-\frac{t}{\tau}} \]

Current starts \( i = i_0 \) and decreases
(a) Graph of current versus time for a charging capacitor

The current decreases exponentially with time as the capacitor charges.
EXALMPE: Consider this circuit:

(a) Capacitor initially uncharged

\[ \varepsilon = 12V; \ C = 0.3 \times 10^{-6}F; \ R = 20 \times 10^3 \Omega \]
Find:

a. The time constant
b. The maximum charge on C
c. The time for the charge to reach 99% of $C_{max}$.
d. The current when $Q = \frac{1}{2} Q_{max}$
e. $I_{max}$
f. $Q$ when $I = 0.2 I_{max}$
DISCHARGING CAPACITOR

When the switch is closed, the charge on the capacitor increases over time while the current decreases.

After long time capacitor has charge $Q_0$.

Connect points $a$ and $c$. 
Now capacitor discharges through the resistor and:

\[ IR = V_c = \frac{Q}{C} \]

\[ I = -\frac{dQ}{dt} \]

\[ -\frac{dQ}{dt} R = \frac{Q}{C} \]

\[ \frac{dQ}{Q} = -\frac{1}{RC} dt \]
Integrate from $t = 0$ to $t = t$ while $O_0$ goes to $Q$.

\[
\int_{Q_0}^{Q} \frac{dQ}{Q} = -\frac{1}{RC} \int_{0}^{t} dt
\]

\[
\ln \frac{Q}{Q_0} = -\frac{t}{RC}
\]

\[
\frac{Q}{Q_0} = e^{-\frac{t}{RC}}
\]

\[
Q = Q_0 e^{-\frac{t}{RC}}
\]

Then

\[
I = -\frac{dQ}{dt} = \frac{Q_0}{RC} e^{-\frac{t}{RC}}
\]

\[
I = \frac{V_0}{R} e^{-\frac{t}{RC}} = I_0 e^{-\frac{t}{RC}}
\]