CHAPTER 24
CAPACITANCE AND DIELECTRICS

MAJOR TOPICS

Calculating Capacitance

Capacitors in Circuits

Energy Storage in Capacitors

Dielectrics
Capacitors store:

Charge

Energy

+Q + - -Q

+ + - -

+ + - -

+ + - -

+BATTERY+ V -
The battery supplies charge to the plates.

The charge $Q$ is proportional to $V$.

\[ Q \propto V \]

Choose the proportionality constant $C$.

\[ Q = CV \]

$C$ is the CAPACITANCE of the capacitor.

\[ C = \frac{Q}{V} \]
If there is charge on a plate we learned in Chapter 21 that there is an electric field $E$. 

\[ +Q \quad + \quad - \quad -Q \]

\[ + \quad P \quad - \]

\[ + \quad - \]

\[ + \quad - \quad BATTERY \]
E at P due to + plate \( E_+ = \frac{\sigma}{2\varepsilon_0} \)

E at P due to - plate \( E_- = \frac{\sigma}{2\varepsilon_0} \)

Both fields point to the right
\[ E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} \]

DEFINITION \[ C = \frac{Q}{V} \]

From Chapter 23

\[ V_b - V_a = -\int E \cdot dl \]

Integrate from negative to positive plate

\[ V_{ab} = -\int Edl\cos 180^0 \]
\[ V_{ab} = \int E \, dl = \int \frac{\sigma}{\varepsilon_0} \, dl \]

\[ \sigma = \frac{Q}{A} \]

\[ V_{ab} = \int \frac{Q}{\varepsilon_0 A} \, dl = \frac{Q}{\varepsilon_0 A} \int dl = \frac{Q \, d}{\varepsilon_0 A} \]

\[ C = \frac{Q}{V_{ab}} = \frac{Q}{\frac{Q \, d}{\varepsilon_0 A}} = \frac{\varepsilon_0 A}{d} \]
Thus for all capacitors \[ C = \frac{Q}{V} \]

And for parallel plate capacitors

\[ C = \frac{\varepsilon_0 A}{d} \]

Parallel plate capacitors are easy:
Area and distance between plates gives C.

If know C then:
Know Q can get V
Know V can get Q
Other geometries are more difficult.

Consider a cylinder inside another one.

\[ C = \frac{Q}{V} \]

We know \( Q = \lambda L \)
Need $V$

$$V = - \int E \cdot dr$$

To get $V$ need $E$ as function of $r$ between cylinders.

Choose Gaussian cylinder of length $l$ where

$r_a < r < r_b$

\[ l \]

\[ r \]

\[ r_a \]

\[ r_b \]
Gauss’s Law

\[ \oint E \cdot dA = \frac{Q_{en}}{\varepsilon_0} \]

\[ Q_{en} = \lambda l \]

\[ \int_{\text{end}} E \cdot dA + \int_{cyl} E \cdot dA + \int_{\text{end}} E \cdot dA = \frac{\lambda l}{\varepsilon_0} \]

\[ \int_{\text{end}} EdA \cos 90^\circ \]

\[ + \int_{cyl} EdA \cos 0^\circ \]

\[ + \int_{\text{end}} EdA \cos 90^\circ = \frac{\lambda l}{\varepsilon_0} \]
\[ 0 + \int_{cyl} E \text{d}A + 0 = \frac{\lambda l}{\varepsilon_0} \]

\[ E2\pi rl = \frac{\lambda l}{\varepsilon_0} \]

\[ E = \frac{\lambda}{2\pi\varepsilon_0 r} \]

Then

\[ V = \int_{r_a}^{r_b} E \cdot \text{d}r = \int_{r_a}^{r_b} E\text{d}r \cos\theta \]
\[ V = \int_{r_a}^{r_b} \frac{\lambda}{2\pi \varepsilon_0 r} \, dr = \frac{\lambda}{2\pi \varepsilon_0} \int_{r_a}^{r_b} \frac{dr}{r} \]

\[ V = \ln \frac{r_b}{r_a} \frac{\lambda}{2\pi \varepsilon_0} \]

\[ C = \frac{Q}{V} = \frac{\lambda L}{\ln \frac{r_b}{r_a} \frac{\lambda}{2\pi \varepsilon_0}} \]

\[ C = \frac{2\pi \varepsilon_0 L}{\ln \frac{r_b}{r_a}} \]
CAPACITORS IN CIRCUITS

Series

(a) Two capacitors in series

**Capacitors in series:**
- The capacitors have the same charge $Q$.
- Their potential differences add:
  \[ V_{ac} + V_{cb} = V_{ab}. \]
(b) The equivalent single capacitor

\[ V = V_1 + V_2 \]

\[ C = \frac{Q}{V} \quad \text{yields} \quad V = \frac{Q}{C} \]

\[ \frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} \]

\[ \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{for series} \]
Parallel

(a) Two capacitors in parallel

Capacitors in parallel:
• The capacitors have the same potential $V$.
• The charge on each capacitor depends on its capacitance: $Q_1 = C_1 V$, $Q_2 = C_2 V$. 

$V_{ab} = V \quad C_1 \quad Q_1 \quad C_2 \quad Q_2$
(b) The equivalent single capacitor

Charge is the sum of the individual charges:

\[ Q = Q_1 + Q_2 \]

Equivalent capacitance:

\[ C_{eq} = C_1 + C_2 \]

\[
Q = Q_1 + Q_2 \\
C = \frac{Q}{V} \quad \text{yields} \\
C \rightarrow Q = CV \\
C_{eq}V = C_1V + C_2V \\
C_{eq} = C_1 + C_2 \text{ for parallel}
\]
ENERGY IN CAPACITOR AND ELECTRIC FIELD

In Chapter 23 we defined \( V = \frac{U}{q} \)

Thus \( \Delta V = \frac{\Delta U}{q} \)

Now charge a capacitor
\[ dW = V \, dq \]
\[ V = \frac{q}{C} \]
\[ dW = \frac{q}{C} \, dq \]
\[ W = \frac{1}{C} \int_0^Q q \, dq = \frac{1}{C} \left[ \frac{q^2}{2} \right]_0^Q \]
\[ W = \frac{1}{2} \frac{Q^2}{C} \]

This is the amount of work to charge the capacitor from 0 charge to a charge of Q.
This is the energy stored in the capacitor.

\[ U = \frac{1}{2} \frac{Q^2}{C} \]

Use definition of capacitance \( C = \frac{Q}{V} \)

\[ U = \frac{1}{2} CV^2 \]

Or \( U = \frac{1}{2} QV \)

Then for parallel plate capacitors
\[ C = \frac{\varepsilon_0 A}{d} \quad \text{and} \quad V = Ed \]

\[ U = \frac{1}{2} CV^2 = \frac{1}{2} \frac{\varepsilon_0 A}{d} E^2 d^2 \]

\[ U = \frac{1}{2} \varepsilon_0 E^2 Ad \]

\[ Ad = \text{volume of capacitor} \]

Energy density \[ u = \frac{U}{V} \]

Therefore \[ u = \frac{U}{V} = \frac{\frac{1}{2} \varepsilon_0 E^2 Ad}{Ad} = \frac{1}{2} \varepsilon_0 E^2 \]
DIELECTRICS

Add material between plates and $C$ increases.

Increases by $K$ the dielectric constant

Parallel Plate Capacitor

\[
C = \varepsilon_0 \frac{A}{d} \quad \text{becomes} \quad K \varepsilon_0 \frac{A}{d}
\]

Define $\varepsilon = K\varepsilon_0$

Then can use $C = \varepsilon \frac{A}{d}$
Other quantities

Energy density

\[ u = \frac{1}{2} \varepsilon_0 E^2 \quad \text{becomes} \quad \frac{1}{2} K\varepsilon_0 E^2 = \frac{1}{2} \varepsilon E^2 \]

Charge on capacitor connected to \( V \).

\[ Q_0 = C_0 V \]

Introduce dielectric
Insert dielectric and $Q$ increases if $V$ remains constant.

\[ C = K C_0 \]

\[ C = K \frac{Q_0}{V} \]

\[ CV = K Q_0 \]

\[ CV = Q = K Q_0 \]
Voltage across capacitor without battery.

\[ C_0 = \frac{Q}{V_0} \]
Adding the dielectric reduces the potential difference across the capacitor.

\[ C = K C_0 \]

\[ C = \frac{Q_0}{V} \]
\[ KC_0 = \frac{Q_0}{V} \]

\[ V = \frac{Q_0}{c_0} \frac{1}{K} \]

But \[ \frac{Q_0}{c_0} = V_0 \]

So \[ V = \frac{V_0}{K} \]

When C not connected to battery inserting dielectric decreases \( V \).
Electric Field in dielectric.

\[ E_D = \frac{V}{d} = \frac{V_0/K}{d} = \frac{V_0/d}{K} = \frac{E_0}{K} \]

EXAMPLE 24.10 is an excellent review of all of these concepts.